9/23 In Class Problems 5– Energy and Collisions

You may find this Binomial Expansion helpful :

$$(1 \pm z)^{-n} = 1 \mp nz + \frac{n(n+1)z^2}{2} \mp \cdots \qquad \text{for } z < 1$$

We have seen that relativistic momentum is $\vec{p} = \gamma m\vec{v}$ and relativistic Energy is $E = \gamma mc^2$.

1. In the limit where $v << c$, what does the relativistic momentum reduce to?

2. In the limit where $v << c$, what does the relativistic energy reduce to?

3. That last one should leave you wondering: why do we pick $E = \gamma mc^2$ to be energy? Well, to see the answer, expand the radical in the γ in a Taylor Series (the binomial expansion.)

   (a) Simplify it. (You should recognize at least one term.) Are there any terms with no v? What is that? Do you remember from Modern?

   (b) What happens now if $v << c$? What about that constant term–does it affect anything in our definition of energy?

   Because of the last question, we define rest energy, $R = mc^2$ and Kinetic Energy, $T = (\gamma - 1)mc^2$.

   Massless particles

4. In classical physics, massless particles would make no sense–at least, they couldn't do anything. Explain. (Do they have energy? momentum? Could they exert forces on other things? Could they do work?)

5. In relativity, there is a loophole. What happens to the relativistic momentum and energy if $m = 0$ and $v = c$? Do any such particles exist? Particles with no mass that always travel with speed c (in vacuum)?

6. Last time you determined that $p_\mu p^\mu = \frac{E^2}{c^2} - p^2 = m^2c^2$. Does that relationship still hold for massless particles? What does it say for massless particles?

7. Photons are massless and travel at speed c. What distinguishes photons of different energy and momenta? (Relativity has no answer, but quantum does! (and you saw it in Modern and probably Chem.))

Collisions

Classically:

1. Mass is conserved
2. Momentum is conserved
3. Kinetic Energy may or may not be conserved
   (a) Elastic collisions conserve KE. Examples: collisions between steel balls, collisions between objects that do not deform.
   (b) Inelastic collisions do not. Examples: objects stick together, objects explode.

Relativistically:

1. Momentum is conserved
2. Energy is conserved
   1 and 2 together mean that the energy-momentum 4-vector is conserved!
3. Kinetic energy and rest energy may or may not be conserved. (One can transform to other, keeping total energy conserved.)
   (a) Elastic collisions conserve KE and RE.
   In particles physics, this means the same particles come out as went in.
   (b) Inelastic collisions do not. Examples: objects stick together (RE and mass increase), objects explode (RE and mass decrease).

When doing these problems, Griffiths expects you to look up the masses on the table near the front of the book.

1. Show that a particle's velocity, $v = \frac{pc^2}{E}$. This can be very useful if you need a particle's speed and you know its energy and momentum.
2. A pion at rest decays into a muon and a neutrino. (Assume the neutrino is massless for all these collision problems.) What is the speed of the muon? Hint 1: Start with conservation of energy. Hint 2: Useful for this problem and many more: if you know the momentum of a particle, use the $\frac{E^2}{c^2} - p^2 = m^2c^2$ invariant to get its energy. (or vice versa)
3. In the lab frame, a proton strikes another proton at rest, and three protons and an anti-proton come out. (This was the way they created the first anti-proton.) What is the threshold energy of the incident proton in the lab frame? Hint: even though it asks for energy in the lab frame, you need to consider the CM frame because only in that frame can you see that the minimum energy for the final four is $4m_pc^2$. Exploit the invariance of m^2c^4 to solve this one.