10/3 In Class Problems 6– Matrix Review

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

1. The determinant
   (a) What is the determinant of B?
   (b) What is the determinant of D?
   (c) What is the determinant of U?
   (d) What is the determinant of σ_x?
   (e) What is the determinant of σ_y?
2. Matrix multiplication
   (a) What is AB?
   (b) What is CD?
   (c) What is $\sigma_x\sigma_y$?
   (d) What is σ_x^2?
   (e) What is σ_y^2?
3. The complex conjugate of a matrix, M, is written M^* and you get it by taking the complex conjugate of each element of the original matrix.
   (a) What is σ_x^*?
   (b) What is σ_y^*?
   (c) What is U^*?
4. The transpose of a matrix, M, is written $\tilde{M}$ and you get it by taking the leaving the diagonal the same, and flipping all the other elements around the diagonal. (Rows become columns.)
   (a) Find $\tilde{\sigma}_y$
   (b) Find $\tilde{U}$

5. The transpose conjugate of a matrix, M, is written $M^\dagger$ (and sometimes $\tilde{M^*}$) and you get it by taking the leaving the diagonal the same, and flipping all the other elements around the diagonal and taking the complex conjugate of each element. (Rows become columns with all elements complex conjugated.)

   (a) Find $B^\dagger$

   (b) Find $\sigma_y^\dagger$

   (c) Find $U^\dagger$

6. A unitary matrix is one where the transpose conjugate of the matrix is its own inverse.

   (a) Find $UU^\dagger$

   (b) Find $U^\dagger U$