10/5 In Class Problems 7 — Rotation and Matrix Review

Rotations

There are two conventions for rotations. One is to rotate the vector itself, and leave the axes unchanged, and the other is to rotate the axes. The first is a bit easier to visualize, so we'll stick with that.

Let's start in 2D:

1. Take a vector $\vec{A}$ and rotate it by θ around the z-axis–the usual convention, measure θ from the x axis.. Call the new vector $\vec{A}'$ How do the components transform? Write it as a matrix so that
$$\vec{A}' = R\vec{A}$$
where R is the rotation matrix.

2. Check your matrix for several special cases:
   (a) If $A = 3\hat{x}$ and $\theta = 90°$ what is A'? Is it what you'd expect?
   (b) If $A = 3\hat{x} + 3\hat{y}$ and $\theta = 45°$ what is A'? Is it what you'd expect?
   (c) You pick one.

3. What is the determinant of R?

4. What is R^2?

5. What is $R^\dagger$?

6. What is $R^\dagger R$? There's a name for this. Do you remember it from Quantum? Or Linear Algebra?

7. What are the eigenvalues of R?

8. What are the corresponding eigenvectors?