10/5 In Class Problems 8 — Angular Momentum

1. Angular momentum, $\boldsymbol{L} \equiv \boldsymbol{r} \times \boldsymbol{p}$. Work out the components of $\boldsymbol{L}$ by doing the cross product by components.

2. Momentum operator:
   Recall from Quantum (or Modern) that we can derive the (Time Independent) Schrödinger Equation by using
   $$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$
   (In this notation, the hat above the momentum indicates an operator, not a unit vector.) Derive the TISE. Start with the total energy, write kinetic in terms of momentum, and then substitute in the momentum operator.

3. Those of you who have taken Quantum may remember the commutator. (Actually, there's at least one problem in 105 as well.) The commutator is defined as:
   $$[A, B] = AB - BA$$
   where A and B are operators. (Recall that operators can be repesented by matrices.) If they commute, the commutator is zero. Show that $[x, p_x] = i\hbar$. Hint: recall there's a trick: you must operate on a function to see that the derivatives don't commute.

   The commutator is important because if two quantities do not commute (such as position and momentum, you cannot find a state that is an eigenstate of both operators. (One or the other, yes, but not both.) It also means there will be an uncertainty relationship between them. Such as the HUP for position and momentum. (For an example proof, see Griffiths, *Introduction to Quantum Mechanics*.)

4. Show that (or argue that) $[x, y] = 0$

5. Show that (or argue that) $[p_x, p_y] = 0$

6. The components of angular momentum do not commute. Find:
   (a) $[L_x, L_y]$
   (b) $[L_y, L_z]$
   (c) $[L_z, L_x]$
   (d) If you replace the x, y, z with i, j, k, do you know how to write one equation to represent all three permutations above?

7. The square of the total angular momentum does commute with each component. Show one that it works for $[L^2, L_z]$. This means $[L^2, \boldsymbol{L}] = 0$.

8. .Define two new operators, L_+ and L_- as
   $$L_\pm = L_x \pm iL_y$$
   . Work out the commutators $[L_z, L_\pm]$.