10/14 In Class Problems 9 — Addition of Angular Momenta and Spin 1/2

Adding angular momenta:

1. You have two electrons, both are spin up. What is the only possible total S_z? What are the possible values of their total spin? Use the Clebsch Gordan coefficients to write the spin state. Does this make sense?
2. You have two particles that are both in the spin state $|20\rangle$. Assume $L = 0$. What values of total spin might you get, and what are the probabilities of each?
3. (A more general case of the first problem.) If you have two spin 1/2 particles, you could possibly add four different combinations. Write out the Clebsch-Gordan decomposition of each of the four possible additions. (Assume there is no orbital angular momentum.)
4. In the last problem, there are two ways to get a total z-component of spin equal to zero. One when the total spin is 1, and one where it's zero. You can solve for the probability of getting either in two ways.
   (a) Method 1: Solve for the states $|10\rangle$ and $|00\rangle$ using (two of) the equations from the last problem. What probabilities do you get for the first particle having been the spin up one? And for the second?
   (b) Method 2: Use the columns of the Clebsch-Gordan coefficients directly! Try reading down the columns instead of across the rows. You should get the same thing. Try it.
5. One particle has $l = 1$ and $s = 1/2$. It could be in the state where its total angular momentum is either 3/2 or 1/2. Given that the total $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ quantum number $j = 3/2$, what are the possible total z-values? List all the possible combinations of m_l and m_s that would yield total angular momentum 3/2. Use the Clebsch Gordan coefficients to find all the probabilities.

   You should be able to answer things like: if the total z-component (of J) is 1/2, what is the probability that m_l was 0? (Please do answer.)

Spin 1/2

For particles of spin 1/2, there are only two possible spin states: we often call them spin up and spin down where m_s is either +1/2 or -1/2. We can have been writing: $\left|\frac{1}{2}\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2}-\frac{1}{2}\right\rangle$ to represent these states. We also have said that we can represent operators, such as spin (S), as matrices. The representations of the components of intrinsic spin as matrices are shown below.

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Using this notation, we want to write the two states as vectors. If we choose

$$\left|\frac{1}{2}\frac{1}{2}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|\frac{1}{2}-\frac{1}{2}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{we can show that:}$$

1. Use matrix/vector multiplication to show that

$$S_z\left|\frac{1}{2}\frac{1}{2}\right\rangle = \frac{\hbar}{2}\left|\frac{1}{2}\frac{1}{2}\right\rangle$$

2. And

$$S_z\left|\frac{1}{2}-\frac{1}{2}\right\rangle = -\frac{\hbar}{2}\left|\frac{1}{2}-\frac{1}{2}\right\rangle$$

(This is how we chose the vector states.)

It is pretty easy to see that a generic state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ can be written as a superposition of the eigenstates of S_z as:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where α^2 is the probably of getting spin up and β^2 is the probability of getting spin down if you measure the z-component of spin on the state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

3. Recall that the components of $\boldsymbol{S}$ do not commute. Recall that means that the same state cannot be an eigenstate of both S_z and S_x. Find the eigenstates of S_x. These must be normalized (meaning they must have magnitude one.)

4. Write the state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ as a superposition of these eigenstates.

5. Using the result of the last problem, what is the chance of getting spin up if you measure the x component of spin?