10/17 In Class Problems 10 — Spin 1/2

We also have said that we can represent operators, such as spin (S), as matrices. The representations of the components of intrinsic spin as matrices are shown below.

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1. What is S_z^2? Show it by matrix multiplication.
2. What is S_y^2? Show it by matrix multiplication.
3. What is the commutator of $[S_x, S_y]$? Work it out and show it with the matrix representation.

For particles of spin 1/2, there are only two possible spin states: we often call them spin up and spin down where m_s is either +1/2 or -1/2. We can have been writing: $\left|\frac{1}{2}\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2} - \frac{1}{2}\right\rangle$ to represent these states. Using this notation, we want to write the two states as vectors. If we choose

$$\left|\frac{1}{2}\frac{1}{2}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|\frac{1}{2} - \frac{1}{2}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ we can show that:

4. Use matrix/vector multiplication to show that

$$S_z\left|\frac{1}{2}\frac{1}{2}\right\rangle = \frac{\hbar}{2}\left|\frac{1}{2}\frac{1}{2}\right\rangle$$

5. And

$$S_z\left|\frac{1}{2} - \frac{1}{2}\right\rangle = -\frac{\hbar}{2}\left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

(This is how we chose the vector states.)

It is pretty easy to see that a generic state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ can be written as a superposition of the eigenstates of S_z as:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where α^2 is the probably of getting spin up and β^2 is the probability of getting spin down if you measure the z-component of spin on the state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

6. Recall that the components of $\boldsymbol{S}$ do not commute. Recall that means that the same state cannot be an eigenstate of both S_z and S_x. Find the eigenstates (eigenvectors) of S_x. These must be normalized (meaning they must have magnitude one.)

7. Write the state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ as a superposition of these eigenstates.

8. Using the result of the last problem, what is the chance of getting spin up if you measure the x component of spin?