Physics 140 Fall 2016

10/24 In Class Problems — Decay Rates and an Intro to Cross Sections

Some definitions:
$\Gamma \equiv$ the decay rate $\equiv$ the probability per unit time that one particle will decay
$\tau \equiv$ the mean lifetime of the particle
$N \equiv$ the total number of particles at any instant in time (t).

1. Given the definition of Γ, it is not hard to see that the number of particles the will decay in a given time interval dt is $N\Gamma dt$. This would decrease the number remaining so that

$$dN = -N\Gamma dt$$

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   (a) Why is there a negative sign in the last equation?

   (b) Solve that DE to find N as a function of time. Assume you start with N_0 particles at time $t = 0$.

2. Example 1: If you start with 1000 particles with a decay rate of 1×10^6 per second, how many will be left in:

   (a) 1 ns

   (b) 1μs

   (c) 1 ms

   (d) 1 s

3. Follow the steps below to show that the mean lifetime of the particle is $\tau = 1/\Gamma$.

   (a) If you start with N_0, how many particles are still around at time t?

   (b) If you start with N_0, how many particles are still around at time $t + dt$?

   (c) Use $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ on the $e^{\Gamma dt}$ term in part (b).

   (d) How many particles decayed from time t to time $t + dt$?

   (e) What fraction decayed from time t to time $t + dt$?

   (f) That fraction can be used as the probability $p(t)$ that an individual selected at random from the initial sample will decay between t and $t + dt$. The mean lifetime is defined as $\tau = \int_0^\infty tp(t)dt$. Use this and your answer from (e) to show that $\tau = 1/\Gamma$.

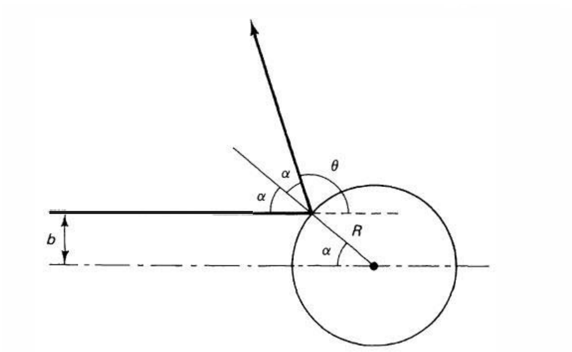
4. Example 2: the muon has a mean lifetime of 2.2 μs. If you start with 1000 muons, how many will be left in 1μs?

Cross Sections

For classical (finite size, $v << c$) particles, if you shoot a particle at a stationary object—the target, the difficulty in hitting the target would just be the area the target presents—the cross-sectional area.

For the figure shown here, b is the impact parameter, the closest distance from the incident particles path to the center of the target.

is the scattering angle.



5. For scattering off a hard sphere of radius R, (imagine marbles, but no rolling) find the relationship between the impact parameter, b and the scattering angle θ. R will be in your answer, but not α.