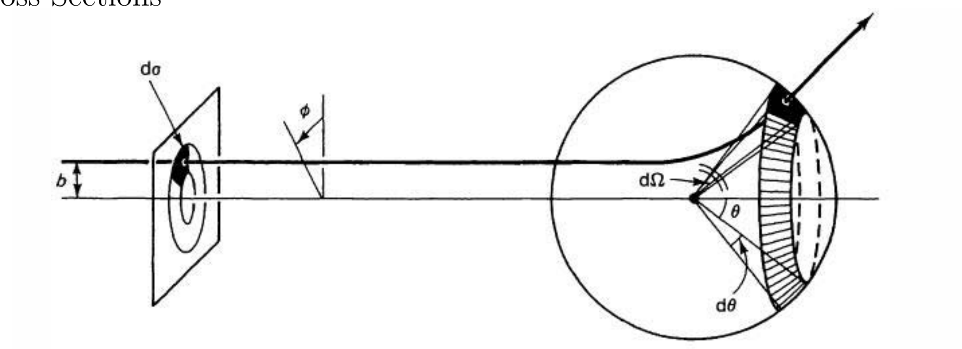
10/28 In Class Problems — Cross Sections

Cross Sections



1. Using the figure above, what is $d\sigma$ (the tiny piece of the area shown) in terms of b and ϕ? (This is basically polar coordinates with different letters.)

2. What is $d\Omega$ in terms of θ and ϕ?

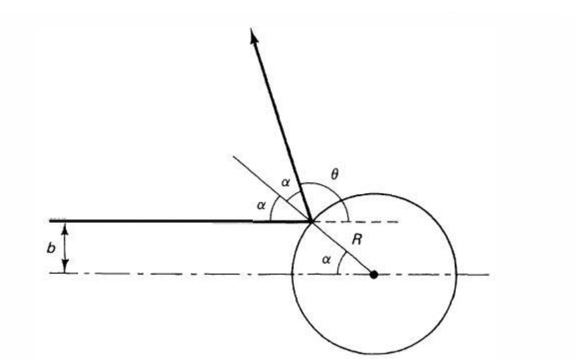
3. The differential cross section, D, is defined as $D = \frac{d\sigma}{d\Omega}$. Write down D using your answers to (1) and (2).

4. Hard Sphere Scattering

   For classical (finite size, $v << c$) particles, if you shoot a particle at a stationary object—the target, the difficulty in hitting the target would just be the area the target presents—the cross-sectional area.

   For the figure shown here, b is the impact parameter, the closest distance from the incident particles path to the center of the target.

   θ is the scattering angle.

   For scattering off a hard sphere of radius R, (imagine marbles, but no rolling), last time you found that $b = R\cos(\theta/2)$

   (a) Find the differential cross section for hard sphere scattering.

(b) Now integrate it to get the total cross section, $\sigma = \int d\sigma = \int D d\Omega$. You should recognize the answer :)

5. Rutherford Scattering: Rutherford scattering occurs when an incident particle of charge q_1 scatters off a target of charge q_2. The target is often much much heavier (the original experiment was an alpha particle incident on a gold nucleus.) This can still be analyzed classically, (depending on the speed of q_1), but the difference between hard sphere and rutherford scattering is that q_1 does not have to hit the target to be deflected.

   Classically, the impact parameter for Rutherford Scattering is

$$b = \frac{q_1 q_2}{2T} \cot(\theta/2)$$

   where T is the kinetic energy of the incident particle.

   (a) Find the differential cross section for Rutherford Scattering.

   (b) Find the total cross section. Did you expect this answer?

6. Luminosity: the luminosity, $\mathcal{L}$ is the number of particles incident per unit area per unit time.

   (a) Write an expression for the number of particles (dN) passing through area $d\sigma$ per unit time:

   (b) Write another expression for dN replacing $d\sigma$ with $D d\Omega$.

   (c) IF the detector covers the entire region where a particle might escape, $N = \mathcal{L}\sigma$ (in words: the event rate is cross section times luminosity). Explain/show this.