10/31 In Class Problems 13— The Delta Function

1. Evaluate $\int_{-\infty}^{\infty}(x^3 - 3x^2 + 2x - 1)\delta(x)dx$
2. Evaluate $\int_{-\infty}^{\infty}(x^3 - 3x^2 + 2x - 1)\delta(x - 2)dx$
3. Evaluate $\int_{-\infty}^{\infty}(x^3 - 3x^2 + 2x - 1)\delta(x + 2)dx$
4. Evaluate $\int_{0}^{\infty}(x^3 - 3x^2 + 2x - 1)\delta(x + 2)dx$
5. Evaluate $\int_{-\infty}^{\infty}(\cos\theta)\delta(\theta - \pi)d\theta$
6. Show that $\delta(kx) = \frac{1}{|k|}\delta(x)$
   (a) Start with $\int_{-\infty}^{\infty} f(x)\delta(kx)dx$ and substitute $y = kx$. Do the integral over y. Do it first with $k > 0$
   (b) Repeat with $k < 0$. (Hint: what happens to the limits?)
   (c) Put the two together.
7. Evaluate $\int_{-\infty}^{\infty}(x^3 - 3x^2 + 2x - 1)\delta(2x)dx$

   $\delta(kx) = \frac{1}{|k|}\delta(x)$ is a special case of a more general rule:

$$\delta(g(x)) = \Sigma_{n=1}^{n} \frac{1}{|g'(x_i)|}\delta(x - x_i)$$

8. Simplify the expression $\delta(x^2 + x - 2)$.
9. Simplify the expression $\delta(\cos x)$.