Phys 140 - QoR
Boyda/Wensley

Homework. The reading is due April 18. Though I haven't assigned any specific writing exercises to accompany it, please prepare specific questions for discussion. We will approach this paper Seminar-style in class on Tuesday. The problems are due April 20.

Reading

- A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of reality be considered complete?" (1935).

Problems

1. Rieffel and Polak 3.2, 3.5, 3.7, 3.8b, and 4.7 (for which you may use the measurement formalism described in class in lieu of projection operators).

2. At the end of class Thursday we considered the entangled two-spin state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x+\rangle_1|x+\rangle_2 + |x-\rangle_1|x-\rangle_2\right).$$

   We computed the probability of measuring the first spin to be up along the z axis, found the resulting state after the wavefunction collapse, and then computed the probabilities for the second spin to be up or down along the z axis. Repeat this process for the product state

$$|\psi'\rangle = |x+\rangle_1|x+\rangle_2.$$

   How do the probabilities for $|\psi'\rangle$ compare to those we found for state $|\psi\rangle$ in class?

3. All points of data analysis requested in the discussion of photon coincidence counting at the output ports of a beamsplitter, attached below.

Photon coincidence counting

We have been interested in testing the proposition that light consists of particles, called photons. A particulate detection should be localized in space and in time, producing a brief burst of current in a photodetector. But we understand from our discussion of the photoelectric effect that a semi-classical description, in which the matter in the detector is quantized and the light is modeled as an electromagnetic wave, is also consistent with such detections.

Another expression of particle nature is this: if a photon is a minimal corpuscle of light, it ought be in some sense indivisible.

Primitively, a beamsplitter is a partially-silvered mirror. It reflects some incident light and allows some light to pass through. For an electromagnetic wave this is simply a process of splitting, in which some of the wave energy travels one direction, some the other. You could think up many other examples where waves behave like this – a wave incident on a knot linking two strings of different density, an ocean wave incident on an underwater ledge, light incident on a pane of glass, etc. A photon incident on a beamsplitter, if indivisible, must go one way or the other. It cannot split in half. Quantum mechanically, it can enter a superposition of transmitted and reflected states, but if we place detectors in the path of transmitted and reflected beams in order to determine which way the photon went, we collapse the wavefunction – we should see a detection in one detector or the other, not both.

Let us begin from our measured quantity: The number of coincident detections in the detectors downstream from the beamsplitter. If there were only ever single photons in the beamsplitter, the number of coincidences ought to be zero.

We know from prior experiments that thermal and laser light sources do not produce photon-number eigenstates, but we believe (and I suppose the result we are working towards is a consistency check on this belief) that the photon pairs created by spontaneous parametric downconversion are entangled two-photon states. We will trigger on one of the two photons of a pair and put the other through the beam splitter. A trigger detection assures us (barring rare accidental coincidences, the effects of which we will attempt to quantify below), a single-photon state is in the beamsplitter.

We have been calling detector A our trigger detector, detectors B and C the detectors placed in the transmitted and reflected beams of the beamsplitter. We want to check how many photons appear simultaneously (within the finite time resolution of our experiment) at B and C given a simultaneous detection of the trigger photon at A. Call this quantity N_{ABC}.

Since the counts N_{ABC} won't be identically zero we want to know at least that they are small relative to something. Relative to what? To the number of detections in detectors B and C separately (but always contingent on the appearance of a trigger photon at A). Therefore we consider $N_{ABC}/(N_{AB}N_{AC})$. And it is convenient to think in terms of fractions of the total number of possible trigger detections N_A:

$$\frac{N_{ABC}/N_A}{(N_{AB}/N_A)(N_{AC}/N_A)}.$$

Each of the fractions above represents our best estimate from data of the *probability* of a coincidence given a single trigger:

$$\frac{N_{ABC}/N_A}{(N_{AB}/N_A)(N_{AC}/N_A)} = \frac{P(B,C|A)}{P(B|A)P(C|A)}.$$

You read the probabilities as follows: $P(B,C|A)$ means the probability of getting detections at B *and* C *given* a detection at A. $P(B|A)$ means the probability of getting a detection at B given a detection at A.

We have related this ratio to a quantity quantum opticians know as the second-order correlation function for the electromagnetic field, which tells how a product of the field at point $\boldsymbol{r}_1$ and time t_1 correlates with the same product at point $\boldsymbol{r}_2$ and time t_2. Letting $\boldsymbol{r}_1$ refer to detector B, $\boldsymbol{r}_2$ to detector C, and studying simultaneous detections ($t_1 = t_2$), we have been denoting this quantity $g^{(2)}_{BC}(0)$, where 0 is the time difference. For the scenario described here,

$$g^{(2)}_{BC}(0) = \frac{P(B,C|A)}{P(B|A)P(C|A)}.$$

This is not a definition but a reduction of the second-order correlation function to our particular experimental circumstances. It is helpful because it ties back to well known theory of the electromagnetic field. I want to emphasize, however, that its meaning for us is precisely that which is embedded in the triple-coincidence counts N_{ABC}: We are checking, given a trigger at A, whether we tend to get detections at B and C at the same time or not.

To be confident in our understanding, we would like a rigorous theory of photons as particles in which the predicted value of this quantity can be seen to be zero. We have it. It is called Quantum Electrodynamics (QED). I started to work out some formalism, by which you could see that the probabilities are inner products and the triple coincidence probability is zero because you never measure three photons from a two photon state. It looks almost tautological – you define a theory of photons and a beamsplitter that preserves photon number – and indeed the resulting inner product is zero. It would be more convincing if I could go on to show you how the formalism explains many more things besides, which in fact include all known processes involving electrons and electromagnetic radiation. But, as Falstaff had it, sometimes discretion is the better part of valor. The QED prediction for $g^{(2)}_{BC}(0)$ for photon number states, along with the predictions for other states of the radiation field and the prediction of the classical theory we went over in class, are listed in Table 1 below.

$g^{(2)}_{BC}(0)$	theory / character of light
0	QED / photon number eigenstates
1	QED / stable coherent (laser) light
2	QED / thermal light (Hanbury-Brown & Twiss result)
≥ 1	Maxwell E&M / electromagnetic waves generally

Table 1: Predictions for the second-order coherence function downstream of a beamsplitter, under various assumptions. Laser and thermal light can equally well be described for this purpose as classical electromagnetic waves.

Data

Along with this assignment is a link to a spreadsheet containing the data Miles and Guy took across eight runs with the same experimental set up. They report 714 data points in total. Each data point contains total counts N_A, N_{AB}, N_{AC}, and N_{ABC} recorded for a time T, integrating the counts over many coincidence counting intervals Δt. The software specifies that $T = 1\,\mathrm{s}$, but on our stopwatches we have found $T \approx 1.1\,\mathrm{s}$. This is a bit of a puzzle, but fortunately we can proceed without resolving it. The quantities Δt and T enter our calculations only in the combination $w = \Delta t/T$, which we can measure. (See below.)

We want to determine best estimates and uncertainties for each of the quantities N_A, N_{AB}, N_{AC}, and N_{ABC}. We expect the individual photons to arrive at the detectors indepedent of those that preceed or follow them, and so the counts ought be distributed according to a Poisson distribution. The best estimate of the count rate is given by the mean, $\langle N\rangle$, and for the Poisson distribution the confidence in this estimate (i.e. the square root of the variance of the mean) is given by $\sqrt{\langle N\rangle/D}$. In this formula D is the total number of data points. The factor $1/\sqrt{D}$ is the usual one dinstinguishing a standard deviation from the standard deviation of the mean: The more data points we have, the more confident we are in the estimation of the mean. Thus we estimate N_A (and the other cases are parallel),

$$N_A = \langle N_A\rangle \pm \sqrt{\frac{\langle N_A\rangle}{D}}.$$

For large $\langle N_A\rangle$, the Poisson distribution is well approximated by a normal distribution, and you could alternately use the more familiar formula for computing the standard deviation of the mean.

Looking at the data, I note that the averages of counts $\langle N_A\rangle$ differ by a few percent across the eight runs. (Each run begins when the counter, leftmost column, resets to -.01.) This could be due to intensity fluctuations in the laser or perhaps due to shifting reflections as people and objects move around the room. Also, the last few counts in each run are as much as ten percent larger than the run average. Could it be an effect of Guy or Miles shifting in getting ready to push the stop button? We might want to compute $g^{(2)}_{BC}(0)$ separately for each of the eight runs and then average, but for simplicity here, let's deal with these effects as follows: Pick the first of the eight runs (lines 2-115), truncate the last several values, those for which the counts N_A have suddenly jumped, and also truncate the first couple of values to be sure we're not picking up a similar effect on the front end. Using these data points and the spreadsheet or a scientific computing platform of your choice, determine N_A, N_{AB}, N_{AC}, and N_{ABC}. Please write some detailed notes or submit your code so I can follow your process.

The uncertainties thus calculated are reflect the natural variations that arise in a Poisson counting process. We worked hard to reduce counts from sources other than the downconversion photons, and by taping over instrument lights, sealing doors, and working in the dark, we achieved zero coincidence counts when the laser was off. When we scatter blue laser light in the room, we get base counts ~ 1100, of which ~ 800 are dark counts generated in the detectors themselves and the rest we believe are red photons generated by fluorescence from the overhead lights and the white paper in the room (which is often treated with optical brightening agents, chemicals that fluoresce). Coincidences remain zero in the presence of scattered blue light. Let's substract off 1100 from N_A to account for this background. (The uncertainty in this value is negligible.)

At this point, you can compute your best estimate for $g^{(2)}_{BC}(0)$.

How do we combine uncertainties? In this case the uncertainties are not independent. A fluctuation of N_A to the large side may increase the other counts as well, in which case you would find some cancellation:

$$
\begin{aligned}
g^{(2)}_{BC}(0) \rightarrow & \frac{(N_A + \delta N_A)(N_{ABC} + \delta N_{ABC})}{(N_{AB} + \delta N_{AB})(N_{AC} + \delta N_{AC})} \\
& \approx \frac{N_A N_{ABC}}{N_{AB} N_{AC}} \left(1 + \frac{\delta N_A}{N_A} + \frac{\delta N_{ABC}}{N_{ABC}} - \frac{\delta N_{AB}}{N_{AB}} - \frac{\delta N_{AC}}{N_{AC}} \right).
\end{aligned}
$$

However, if you look at some data points you'll see the relationship between fluctuations in the different channels is not that simple. If we were talking about independent errors, normally distributed, the thing to do is to add fractional uncertainties in quadrature. In this case, I'm not sure. Conservatively, we could simply add all the fractional uncertainties. Or, observing that the fractional uncertainty in N_{ABC} is an order of magnitude larger than the fractional uncertainties in the the others, we could simply ingnore the others. Let's do that. Assign to $g^{(2)}_{BC}(0)$ the same fractional uncertainty as you find for N_{ABC}. Give a final result for $g^{(2)}_{BC}(0)$, including (absolute) uncertainty, and use it to rule out a classical Maxwellian interpretation of the experiment.

You could also reasonably compute $g^{(2)}_{BC}(0)$ for each data point separately, average those results, and take the uncertainty as the standard deviation of the mean. Do this and compare to the procedure above – the numbers should work out to be very close, both for the mean and the uncertainty.

Let it be said, in these machinations we are wrestling over fractions of a percent. Since we don't know what we don't know – what we can and can't afford to ignore – it's worth thinking carefully. I'd like to hear any alternate ideas you have for handling the data.

You should have found a value for $g^{(2)}_{BC}(0)$ much less than one, but not zero. Let us try to understand where the triple counts came from.

For this we will need to know the coincidence interval in the form $w = \Delta t/T$. We measure w by using scattered light (so as not to overwhelm the detectors) from a stable, coherent laser source. In that case we know $g^{(2)}_{AB}(0) = 1$. (We would need to know the properties of the source in more detail than we do to put an uncertainty on this. Instead we are going to average across two of sources.) Now we are counting coincidences between detectors A and B, with no trigger and no beamsplitter. For a Poisson distribution with average counts N integrated for time T, the probability of getting a single count in interval Δt is N divided by the number of intervals, $N/(T/\Delta t) = Nw$, so long as Nw is small. Therefore,

$$1 = g^{(2)}_{AB}(0) = \frac{P(A,B)}{P(A)P(B)} = \frac{N_{AB}w}{(N_A w)(N_B w)}.$$

By measuring N_A, N_B, and N_{AB}, we can determine w. The blue laser causes fluorescence in the room (producing light for which $g^{(2)}_{AB}(0) \neq 1$), and we don't have a laser in red or green that is intensity stabilized. Our HeNe laser oscillates in intensity, but on a time scale longer than T, so we can reasonably compute w from each data point separately and then average. The diode lasers we use in the Phys 2/4 labs are also reasonably stable. Across five runs with HeNe and red diode laser, the results were consistent with each other to within calculated uncertainties, from which I estimate

$$w = 9.1 \pm 0.1 \cdot 10^{-9}.$$

That means if our value of $T = 1.1\,\mathrm{s}$ is correct, the coincidence interval is ten nanoseconds.

We could subtract a background for $g^{(2)}_{BC}(0)$, but let's think about triple counts directly. One way we could get a triple count that doesn't involve a photon splitting on the beamsplitter would be to get a coincidence of two photons in the AB channel, plus a random third photon in detector C (e.g. from another downconversion pair.) The probability of getting a random C photon in any given interval is $N_C w$, and we have N_{AB} intervals with AB coincidences, so the total expected triple coincidences is $N_{AB} \times N_C w$. Likewise, we could get a coincidence in the AC channel with a random B photon. We expect $N_{AC}N_B w$ of those. These are disjoint possibilities, so we can add them. We might also get three independent photons in channels A, B, and C, but the likelihood of that is negligible. In other words, our expected background is

$$N_{ABC}(bkgd) = N_{AB}N_C w + N_{AC}N_B w.$$

The largest fractional uncertainty here is that in w, about one percent. After computing $N_{ABC}(bkgd)$, subtract it from your value for N_{ABC} determined above. What do you find?

Yep, I don't get zero either, within calculated uncertainties. There is an important effect we have not accounted for. What to do? Our fibers were bad, so we've replaced them. Check our theory. Align and run again.