Phys 140 - QoR
Boyda/Wensley

Bell homework. The reading is for Tuesday, May 2, and the rest is due Thursday, May 4.

Reading

- Greenstein and Zajonc §6.1 − 6.4.

Short answer

1. What does the condition of *locality* require of a physical theory?
2. According to Einstein, Podolsky, and Rosen, what does it mean for something to be *real*, that is to say, a part of our physical reality?
3. What is a hidden variable?
4. Explain the meaning and import of Bell's inequality as you would to a close friend or grandparent (neither of whom, for the purposes of this excercise, may be a physicist).

Problems

The first set of problems refer to Bell's inequality in the form

$$1 + E(\hat{b}, \hat{c}) \geq |E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{c})|. \tag{1}$$

The system is two spin-one-half particles created with spins opposing, to be measured with two spatially separated Stern-Gerlach detectors A and B with orientations chosen from unit vectors $\hat{a}$, $\hat{b}$, $\hat{c}$.

1. Show that when $\hat{a}$ and $\hat{c}$ lie 60° either side of $\hat{b}$, the quantum mechanical expectation values for the entangled state

$$|\psi\rangle_{entangled} = \frac{1}{\sqrt{2}} \left(|z+\rangle_A |z-\rangle_B - |z-\rangle_A |z+\rangle_B \right)$$

   violate the above inequality. Recall from class that the expectation value in this state is given by

$$E(\hat{a}, \hat{b}) = -\cos\phi = -\hat{a} \cdot \hat{b}, \tag{2}$$

   where ϕ is the angle between $\hat{a}$ and $\hat{b}$.

2. By contrast, show that the naive hidden variable theory presented in Bell's paper or in G&Z §5.3 satisfies the inequality for this scenario.

3. Assuming the detectors are oriented with $\hat{b}$ in the middle and $\hat{a}$ and $\hat{c}$ at equal angles ϕ to the left and right of $\hat{b}$, find the set of all angles ϕ for which the quantum mechanical expectation values do *not* violate the above inequality.

The goal of this next problem is to see whether we can generate a violation of the above inequality with a quantum mechanical product state $|\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$. Given a state $|\psi\rangle$, the expectation value $E(\hat{a}, \hat{b})$ can be computed directly in quantum mechanics as

$$E(\hat{a}, \hat{b}) = \langle\psi|(\vec{\sigma}_A \cdot \hat{a})(\vec{\sigma}_B \cdot \hat{b})|\psi\rangle.$$

Here $\vec{\sigma}_A$ is the vector of Pauli matrices acting on the state of the particle at detector A, and $\vec{\sigma}_B$ similarly for the particle at detector B.

4. First show, almost trivially, that the quantum mechanical expectation value can be written as a product of expectation values, where one multiplicand depends only on the state $|\psi\rangle_A$ and the detector setting at A, and the other multilpicand depends only on the state $|\psi\rangle_B$ and the detector setting at B. Product states thus satisfy the condition of locality as laid out by Bell.

If detector settings $\hat{a}$, $\hat{b}$, $\hat{c}$, can be any unit vectors on the sphere, then without loss of generality we can let our product state of opposed spins be

$$|\psi\rangle = |z+\rangle_A |z-\rangle_B.$$

Furthermore, as the expectation value does not depend on choice of basis, we can work in the $|z\pm\rangle$ basis.

5. Compute the expectation value

$$\langle\psi|(\vec{\sigma}_A \cdot \hat{a})(\vec{\sigma}_B \cdot \hat{b})|\psi\rangle$$

   for this choice of $|\psi\rangle$. How does this expectation value compare to the result (2) for the entangled state?

6. Show there do not exist detector settings $\hat{a}$, $\hat{b}$, $\hat{c}$ for which spins in the product state violate the inequality (1). I did it numerically, but maybe one of you can come up with a proof.

This result suggests that entanglement is essential to generate violations of Bell's inequalities.

The remaining problems come from Dean Wensley:

4. Inequality Derivation: In class I introduced the experiment of sending a photon through a polarizer set at different angles. Using classical statistics, I derived the following inequality:

$$n(a, \bar{b}) + n(b, \bar{c}) \geq n(a, \bar{c}),$$

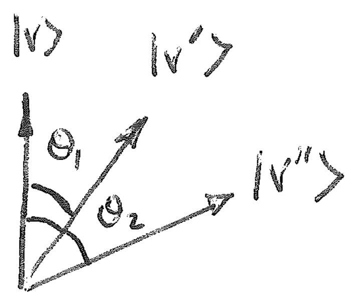
where $n(a, \bar{b})$ is the number of photons (out of a total of N sent through the polarizer) that would pass through polarizer a, but not polarizer b.

Using the same classical analysis, develop an inequality using $n(\bar{a}, c)$ and $n(b, \bar{c})$.

5. General Case of Two Polarizers:. In class I derived the probabilities for photons in the entangled state

$$\psi = \frac{1}{\sqrt{2}} \left(|h\rangle|h\rangle + |v\rangle|v\rangle \right)$$

passing/not passing through two different polarizers. I set one of the polarizers along the vertical axis and the other polarizer at an angle ϕ with respect to the vertical. Derive the probabilities for one polarizer set at θ_1 with respect to the vertical and the other set at θ_2 with respect to the vertical. Show that you get the same probability functions I derived with ϕ replaced by $\theta_2 - \theta_1$.



6. Quantum Mechanics and Classical Inequalities. In this problem you will check the classically derived inequality using the quantum theory for two different sets of polarizer settings:

$$n(a, \bar{b}) + n(b, \bar{c}) \geq n(a, \bar{c}).$$

(a) Use $a = 0°, b = 22.5°$, and $c = 45°$.
(b) Use $a = 0°, b = 45°$, and $c = 90°$.