Short answer

1. In his devilish proposal for testing the limits of quantum mechanics on a cat, Schrödinger concocts a mechanism by which we can imagine establishing a superposition of macroscopic physical variables, being, in the case of the cat, life and death. Why do we never observe macroscopic superpositions in our daily lives?

   The answer is environmental decoherence. Macroscopic systems are inevitably and irrevocably coupled to large ambient Hilbert spaces of atoms, photons, etc. In the absence of special mechanisms to protect quantum coherence, interactions with this environment quickly – much faster than any other dynamics of the system – turn any macroscopic superposition into a mixture. A mixture represents a situation where one of the possibilities of the original superposition has been realized, although we as observers may not yet have determined which one.

Handwritten solutions follow, but first, comments on two of the problems:

1. Almost any setting of the SG apparatus will allow you to distinguish statistically between the pure and mixed states, in that the average numbers of spins measured up and down will differ. However, depending on your choice, with only one hundred spins measured, natural statistical variations might dwarf the expected differences between pure and mixed states.

   On the other hand, if you choose to put the apparatus along the pure state $|\psi\rangle$, then every pure state spin will be measured spin up along that axis. A single spin-down measurement implies you are measuring from the mixed state. That's the cleanest sort of measurement you can image and the one Prof. Wensley indicates in his solution.

   Another decent choice some of you made was to measure in the $|x\pm\rangle$ basis. Then you would compute the probabilities of spin-up / spin-down measurements and show that they differ for the two boxes:

   For the pure state $|\psi\rangle = \frac{1}{2}|z+\rangle - \frac{\sqrt{3}}{2}|z-\rangle$,

$$P(\hat{S}_x \to +\frac{\hbar}{2}) = |\langle x+|\psi\rangle|^2 = |\frac{1}{2\sqrt{2}} - \frac{3}{2\sqrt{2}}|^2 = \frac{1}{2} - \frac{\sqrt{3}}{4}$$
$$P(\hat{S}_x \to -\frac{\hbar}{2}) = |\langle x-|\psi\rangle|^2 = |\frac{1}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}|^2 = \frac{1}{2} + \frac{\sqrt{3}}{4}$$

   For the mixed state, you can either use the density matrix

$$\rho = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{3}{4} \end{pmatrix},$$

or you can tabulate probabilities. I'll do the latter. With 1/4 probability you have the state $|z+\rangle$, for which

$$P(\hat{S}_x \to +\frac{\hbar}{2}) = |\langle x + |z+\rangle|^2 = \frac{1}{2}$$
$$P(\hat{S}_x \to -\frac{\hbar}{2}) = |\langle x - |z+\rangle|^2 = \frac{1}{2}.$$

With 3/4 probability you have the state $|z-\rangle$, for which

$$P(\hat{S}_x \to +\frac{\hbar}{2}) = |\langle x + |z-\rangle|^2 = \frac{1}{2}$$
$$P(\hat{S}_x \to -\frac{\hbar}{2}) = |\langle x - |z-\rangle|^2 = \frac{1}{2}.$$

Putting these results together,

$$P(\hat{S}_x \to +\frac{\hbar}{2}) = \frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2} = \frac{1}{2}$$
$$P(\hat{S}_x \to -\frac{\hbar}{2}) = \frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2} = \frac{1}{2}.$$

A measurement in the $|z\pm\rangle$ basis will not allow you to distinguish between the two boxes. For the pure state $|\psi\rangle = \frac{1}{2}|z+\rangle - \frac{\sqrt{3}}{2}|z-\rangle$,

$$P(\hat{S}_z \to +\frac{\hbar}{2}) = |\langle z + |\psi\rangle|^2 = |\frac{1}{2}|^2 = \frac{1}{4}$$
$$P(\hat{S}_z \to -\frac{\hbar}{2}) = |\langle z - |\psi\rangle|^2 = |-\frac{\sqrt{3}}{2}|^2 = \frac{3}{4}$$

On the face of it these are the same probabilities you get for the mixed state.

4. If you reliquish information about the second particle, both reduced density matrices represent a classical mixture of possibilities for the first particle, 50/50 $|0\rangle$ or $|1\rangle$. The differences between the two Bell states lie in the different correlations between the particles.

(1)

mixture : $\frac{1}{4}$ in $|z+\rangle$, $\frac{3}{4}$ in $|z-\rangle$

pure : $|\psi\rangle = \frac{1}{2}|z+\rangle - \frac{\sqrt{3}}{2}|z-\rangle$

→ No matter how the SG is oriented for the mixed states there will be two types of spins observed.

→ For the pure state there will be an orientation of the SG where only one type of spin is observed. This will be when the SG is oriented so $|\psi\rangle$ is one of the basis vectors in the x-z plane

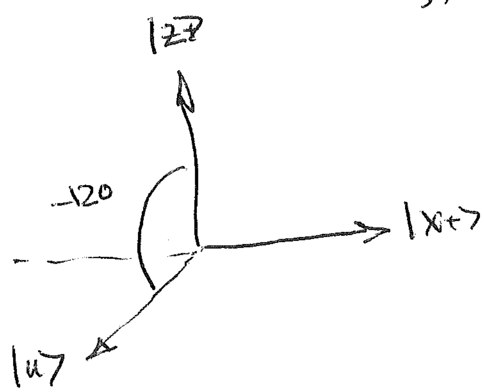
I can write the general form of a pure spin state in x-z :

$$|u\rangle = \cos\frac{\theta}{2}|z+\rangle + \sin\frac{\theta}{2}|z-\rangle$$

For the state $|\psi\rangle$:

$$\cos\frac{\theta}{2} = \frac{1}{2} \quad \rightarrow \quad \theta = -120$$

$$\sin\frac{\theta}{2} = -\frac{\sqrt{3}}{2}$$



(2)

For the mixed state:

$$\hat{\rho}_M = \sum_{i=1}^{2} P_i \, |z\pm\rangle\langle z\pm|$$

$$\boxed{\hat{\rho}_M = \frac{1}{4}|z+\rangle\langle z+| + \frac{3}{4}|z-\rangle\langle z-|}$$

For the pure state:

$$\hat{\rho}_P = \left(\frac{1}{2}|z+\rangle - \frac{\sqrt{3}}{2}|z-\rangle\right)\left(\frac{1}{2}\langle z+| - \frac{\sqrt{3}}{2}\langle z-|\right)$$

$$\boxed{\begin{aligned}\hat{\rho}_P = \frac{1}{4}|z+\rangle\langle z+| &- \frac{\sqrt{3}}{4}|z+\rangle\langle z-| \\ &- \frac{\sqrt{3}}{4}|z-\rangle\langle z+| + \frac{3}{4}|z-\rangle\langle z-|\end{aligned}}$$

Matrix representations:

$$\rho_m = \frac{1}{4}\begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \frac{3}{4}\begin{pmatrix}0\\1\end{pmatrix}(0\ 1)$$

$$= \frac{1}{4}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \frac{3}{4}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

$$\boxed{\rho_m = \begin{pmatrix}1/4 & 0\\0 & 3/4\end{pmatrix}}$$

2 cont

$$\rho_p = \frac{1}{4}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} + \frac{3}{4}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} - \frac{\sqrt{3}}{4}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} - \frac{\sqrt{3}}{4}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \frac{3}{4}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} - \frac{\sqrt{3}}{4}\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} - \frac{\sqrt{3}}{4}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}$$

$$\Rightarrow \boxed{\rho_p = \begin{pmatrix}1/4 & -\frac{\sqrt{3}}{4}\\ -\frac{\sqrt{3}}{4} & 3/4\end{pmatrix}}$$

③ $\langle S_x \rangle = Tr(S_x \hat{\rho})$ $\qquad S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Mixed:

$$\langle S_x \rangle = Tr\ \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2} Tr\begin{pmatrix} 0 & 3/4 \\ 1/4 & 0 \end{pmatrix}$$

$$= 0$$

$$P(S_x = +\tfrac{\hbar}{2}) = Tr(\mathbb{P}_{|x+\rangle} \rho_m)$$

$$= Tr\ |x+\rangle\langle x+| \left(\tfrac{1}{4}|z+\rangle\langle z+| + \tfrac{3}{4}|z-\rangle\langle z-|\right)$$

$$= Tr\left(\tfrac{1}{4}|x+\rangle\langle x+|z+\rangle\langle z+| + \tfrac{3}{4}|x+\rangle\langle x+|z-\rangle\langle z-|\right)$$

$$= Tr\left(\tfrac{1}{4}|x+\rangle\left(\tfrac{1}{\sqrt{2}}\right)\langle z+| + \tfrac{3}{4}|x+\rangle\left(\tfrac{1}{\sqrt{2}}\right)\langle z-|\right)$$

compute $|z\pm\rangle$ basis:

$$= \frac{1}{4\sqrt{2}}\langle z+|x+\rangle\langle z+|z+\rangle + \frac{3}{4\sqrt{2}}\langle z+|x+\rangle\langle z-|z+\rangle$$

$$+ \frac{1}{4\sqrt{2}}\langle z-|x+\rangle\langle z+|z-\rangle + \frac{3}{4\sqrt{2}}\langle z-|x+\rangle\langle z-|z-\rangle$$

$$= \frac{1}{4\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + 0 + 0 + \frac{3}{4\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{8} + \frac{3}{8} = \boxed{\frac{1}{2}}$$

3 cont

Pure:

$$\langle S_x \rangle = \mathrm{Tr}\ \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & 3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\ \mathrm{Tr}\begin{pmatrix} -\frac{\sqrt{3}}{4} & 3/4 \\ 1/4 & -\sqrt{3}/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(-\frac{2\sqrt{3}}{4}\right)$$

$$\boxed{\langle S_x \rangle = -\frac{\hbar\sqrt{3}}{4}}$$

$$P\left(S_x = +\frac{\hbar}{2}\right) = \mathrm{Tr}\left(\hat{\mathbb{P}}_{|x+\rangle}\ \hat{\rho}_p\right)$$

$$\hat{\mathbb{P}}_{|x+\rangle} = |x+\rangle\langle x+|$$

$$\rightarrow \mathbb{P}_{|x+\rangle} = \left(\frac{1}{\sqrt{2}}\right)^2\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$P\left(S_x = +\frac{\hbar}{2}\right) = \mathrm{Tr}\ \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & 3/4 \end{pmatrix}$$

$$= \frac{1}{2}\ \mathrm{Tr}\begin{pmatrix} \frac{1}{4}-\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4}+\frac{3}{4} \\ 1/4-\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4}+3/4 \end{pmatrix}$$

$$= \frac{1}{2}\left(\frac{1}{4}-\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}+\frac{3}{4}\right) = \frac{1}{8}\left(4-2\sqrt{3}\right)$$

$$\Rightarrow \boxed{P\left(S_x = +\frac{\hbar}{2}\right) = \frac{1}{2}-\frac{\sqrt{3}}{4}}$$

3 cont

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Mixed:

$$\langle S_z \rangle = \mathrm{Tr}\, \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\,\mathrm{Tr}\begin{pmatrix} 1/4 & 0 \\ 0 & -3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(-\frac{1}{2}\right)$$

$$\boxed{\langle S_z \rangle = -\frac{\hbar}{4}}$$

$$P\left(S_z = +\frac{\hbar}{2}\right) = \mathrm{Tr}\left(\hat{\mathbb{P}}_{|z+\rangle}\,\hat{\rho}\right)$$

$$|z+\rangle\langle z+| \Rightarrow \mathbb{P}_{|z+\rangle} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \mathrm{Tr}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/4 & 0 \\ 0 & -3/4 \end{pmatrix}$$

$$= \mathrm{Tr}\begin{pmatrix} 1/4 & 0 \\ 0 & 0 \end{pmatrix} = \boxed{1/4}$$

3cont

Pure:

$$\langle S_z \rangle = \mathrm{Tr}\ \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1/4 & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & 3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\ \mathrm{Tr}\begin{pmatrix} 1/4 & -\sqrt{3}/4 \\ \sqrt{3}/4 & -3/4 \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(\frac{1}{4} - \frac{3}{4}\right)$$

$$\boxed{\langle S_z \rangle = -\frac{\hbar}{4}}$$

$$P\left(S_z = +\frac{\hbar}{2}\right) = \mathrm{Tr}\left(\hat{\mathbb{P}}_{|z+\rangle}\hat{\rho}\right)$$

$$= \mathrm{Tr}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/4 & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & 3/4 \end{pmatrix}$$

$$= \mathrm{Tr}\begin{pmatrix} 1/4 & -\sqrt{3}/4 \\ 0 & 0 \end{pmatrix}$$

$$\boxed{P\left(S_z = +\frac{\hbar}{2}\right) = \frac{1}{4}}$$

4

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B \right)$$

$$\hat{\rho}_- = |\psi^-\rangle\langle\psi^-| = \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B \right)\frac{1}{\sqrt{2}}\left(\langle 0|_B \langle 0|_A - \langle 1|_B \langle 1|_A \right)$$

$$= \frac{1}{2}\Big(|0\rangle_A |0\rangle_B \langle 0|_B \langle 0|_A - |0\rangle_A |0\rangle_B \langle 1|_B \langle 1|_A - |1\rangle_A |1\rangle_B \langle 0|_B \langle 0|_A$$
$$+ |1\rangle_A |1\rangle_B \langle 1|_B \langle 1|_A \Big)$$

$$\mathrm{Tr}_B\, \hat{\rho}_- = \langle 0|_B \psi^-\rangle\langle\psi^-|0\rangle_B + \langle 1|_B \psi^-\rangle\langle\psi^-|1\rangle_B$$

$$= \frac{1}{2}\Big(|0\rangle_A \langle 0|0\rangle_B \langle 0|0\rangle_B \langle 0|_A - |0\rangle_A \langle 0|0\rangle_B \cancelto{0}{\langle 1|0\rangle_B} \langle 1|_A$$
$$- |1\rangle_A \cancelto{0}{\langle 0|1\rangle_B} \langle 0|_A \langle 0|0\rangle_B + |1\rangle_A \langle 0|0\rangle_B \cancelto{0}{\langle 1|0\rangle_B} \langle 1|_A \Big)$$
$$+ \frac{1}{2}\Big(|0\rangle_A \cancelto{0}{\langle 1|0\rangle_B} \langle 0|1\rangle_B \langle 0|_A - |0\rangle_A \cancelto{0}{\langle 1|0\rangle_B} \langle 1|1\rangle_B \langle 1|_A$$
$$- |1\rangle_A \langle 1|1\rangle_B \cancelto{0}{\langle 0|1\rangle_B} \langle 0|_A + |1\rangle_A \langle 1|1\rangle_B \langle 1|1\rangle_B \langle 1|_A \Big)$$

$$= \boxed{\frac{1}{2}\left(|0\rangle_A \langle 0|_A + |1\rangle_A \langle 1|_A \right)}$$

(4 cont)

$$\mathrm{Tr}_A\, \hat{\rho}_- = {}_A\langle 0|\psi^-\rangle\langle\psi^-|0\rangle_A + {}_B\langle 1|\psi^-\rangle\langle\psi^-|1\rangle_B$$

$$= \frac{1}{2}\Big({}_A\langle 0|0\rangle_A\, |0\rangle_B\, {}_B\langle 0|\, {}_A\langle 0|0\rangle_A - {}_A\langle 0|0\rangle_A\, |0\rangle_B\, {}_B\langle 1|\, \cancelto{0}{{}_A\langle 1|0\rangle_A}$$

$$- \cancelto{0}{{}_A\langle 0|1\rangle_A}\, |1\rangle_B\, {}_B\langle 0|\, {}_A\langle 0|1\rangle_A + \cancelto{0}{{}_A\langle 0|1\rangle_A}\, |1\rangle_B\, {}_B\langle 1|\, \langle 1|0\rangle_A \Big)$$

$$+ \frac{1}{2}\Big(\cancelto{0}{\langle 1|0\rangle_A}\, |0\rangle_B\, {}_B\langle 0|\, \langle 0|1\rangle_A - \cancelto{0}{\langle 1|0\rangle_A}\, |0\rangle_B\, {}_B\langle 1|\, \langle 1|1\rangle_A$$

$$- \langle 1|1\rangle_A\, |1\rangle_B\, {}_B\langle 0|\, \cancelto{0}{\langle 0|1\rangle_A} + \langle 1|1\rangle_A\, |1\rangle_B\, {}_B\langle 1|\, \langle 1|1\rangle_A \Big)$$

$$= \boxed{\frac{1}{2}\left(|0\rangle_B\, {}_B\langle 0| + |1\rangle_B\, {}_B\langle 1| \right)}$$

4 cont

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right)$$

$$\hat{\rho}_+ = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right) \cdot \frac{1}{\sqrt{2}}\left(\langle 1|_B \langle 0|_A + \langle 0|_B \langle 1|_A\right) \equiv |\Phi^+\rangle\langle\Phi^+|$$

$$= \frac{1}{2}\Big(|0\rangle_A |1\rangle_B \langle 1|_B \langle 0|_A + |0\rangle_A |1\rangle_B \langle 0|_B \langle 1|_A + |1\rangle_A |0\rangle_B \langle 1|_B \langle 0|_A$$
$$+ |1\rangle_A |0\rangle_B \langle 0|_B \langle 1|_A\Big)$$

$$\mathrm{Tr}_B\, \hat{\rho}_+ = \langle 0|_B \psi^-\rangle\langle\psi^-|0\rangle_B + \langle 1|_B \psi^-\rangle\langle\psi^-|1\rangle_B$$

$$= \frac{1}{2}\Big(|0\rangle_A \cancelto{0}{\langle 0|1\rangle_B} \langle 1|0\rangle_B \langle 0|_A + |0\rangle_A \cancelto{0}{\langle 0|1\rangle_B} \langle 0|0\rangle_B \langle 1|_A$$
$$+ |1\rangle_A \cancelto{0}{\langle 0|0\rangle_B} \langle 1|0\rangle_B \langle 0|_A + |1\rangle_A \langle 0|0\rangle_B \langle 0|0\rangle_B \langle 1|_A\Big)$$

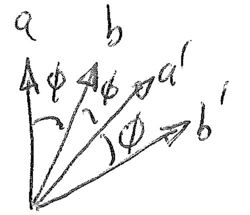
$$+ \frac{1}{2}\Big(|0\rangle_A \langle 1|1\rangle_B \langle 1|1\rangle_B \langle 0|_A + |0\rangle_A \langle 1|1\rangle_B \cancelto{0}{\langle 0|1\rangle_B} \langle 1|_A$$
$$+ |1\rangle_A \cancelto{0}{\langle 1|0\rangle_B} \langle 1|1\rangle_B \langle 0|_A$$
$$+ |1\rangle_A \cancelto{0}{\langle 1|0\rangle_B} \langle 0|1\rangle_B \langle 1|_A\Big)$$

$$= \boxed{\frac{1}{2}\left(|1\rangle_A \langle 1|_A + |0\rangle_A \langle 0|_A\right)}$$

By symmetry → just switch $A \leftrightarrow B$, the same result is obtained for B:

$$\mathrm{Tr}_A\, |\Phi^+\rangle\langle\Phi^+| = \boxed{\frac{1}{2}\left(|1\rangle_B \langle 1|_B + |0\rangle_B \langle 0|_B\right)}$$

(5)



For entangled photon state:

$$E(a,b) = \cos 2\phi$$

The CHSH inequality for $\phi = 60^\circ$ is:

$$S = |E(a,b) - E(a,b') + E(a',b) + E(a',b')|$$

$$= |\cos 120^\circ - \cos 360^\circ + \cos 120^\circ + \cos 120^\circ|$$

$$= |3\cos 120^\circ - \cos 360^\circ|$$

$$= |-1.5 - 1|$$

$= 2.5 > 2$, so the inequality is violated.

(6) For the spin state $\frac{1}{\sqrt{2}}\left(|z+\rangle|z+\rangle + |z-\rangle|z-\rangle\right)$,

$$E(a,b) = -\cos\phi$$

For $\phi = 22.5$

$$S = \left|-\cos 22.5^\circ + \cos(3(22.5^\circ)) - \cos 22.5^\circ - \cos 22.5^\circ\right|$$

$$= \left|-3\cos 22.5^\circ + \cos 67.5^\circ\right|$$

$= 2.31 > 2$, so the inequality is violated

The HVT expectation is:

$$E(\phi) = \frac{2\phi}{\pi} - 1$$

$$\Rightarrow S = \left|\left(\frac{2\phi}{\pi} - 1\right) - \left(\frac{2(3\phi)}{\pi} - 1\right) + \left(\frac{2\phi}{\pi} - 1\right) + \left(\frac{2\phi}{\pi} - 1\right)\right|$$

$$= \left|3\left(\frac{2\phi}{\pi} - 1\right) - \left(\frac{6\phi}{\pi} - 1\right)\right|$$

$$= \left|\cancel{\frac{6\phi}{\pi}} - 3 - \cancel{\frac{6\phi}{\pi}} + 1\right|$$

$= 2 \leq 2$, so the inequality is satisfied.