Physics 140 QoR - Final exam
May 21-25, 2017

You may use any inanimate resource in answering the questions below, including texts, computers, and the internet. You may not discuss the exam with any djinn, fairy, or human. As a matter of trust, this is to be your own work. Responses are due at 8:30 a.m. on Thursday, May 25. Good luck!

I. Short short answer

1. What is a quantum superposition? How does a superposition differ from a classical mixture?
2. What characteristics make a quantum state a "cat" state?

II. Longer short answer

1. We performed three experiments on photon pairs created by spontaneous parametric downconversion in BBO crystals. Choose one of the two experiments for which you were not primarily responsible. Explain what the experiment was designed to demonstrate, the experimental configuration, and the outcome (the actual outcome reported by your classmates, in as much detail as you remember) of the experiment.
2. Our course is titled "Questions of Reality." Explain.

III. Computation

1. For each of the electron spin states below, specify a basis in which the state is a superposition of basis states. Specify a basis in which the state is not a superposition.
   (a) $|\psi\rangle = |z+\rangle$
   (b) $|\psi\rangle = \sqrt{\frac{1}{3}}|z+\rangle - i\sqrt{\frac{2}{3}}|z-\rangle)$
2. You have four polarizers set a fixed angles, one at 0°, two at 30°, and one at 60° relative to the vertical direction in the room. You will place them in sequence in a beam of intially unpolarized light.
   (a) In what order should you place the four polarizers so that the most possible light passes through? What is the probability any given photon passes through?
   (b) In what order should you place the four polarizers so that the least possible light passes through? What is the probability any given photon passes through?

3. Are the following states entangled? If so, prove it. If not, give a product-state decomposition.

   (a) $|\psi\rangle = \frac{1}{\sqrt{3}}(|z+\rangle|x+\rangle + |z+\rangle|x-\rangle + |z-\rangle|z+\rangle)$

   (b) $|\psi\rangle = \frac{1}{\sqrt{3}}(|z+\rangle|x+\rangle - |z+\rangle|x-\rangle + |z-\rangle|z+\rangle)$

4. An electron is prepared in the state

$$|\psi\rangle = \cos\frac{\pi}{8}|y+\rangle - \sin\frac{\pi}{8}|y-\rangle.$$

   (a) What is the density operator (in bra-ket notation) corresponding to this state?

   (b) What is the matrix representation of the density operator in the $|z\pm\rangle$ basis?

   (c) Using the density matrix, compute $\langle S_y\rangle$ and $\langle S_z\rangle$

5. For the following question you will need the posted data from our Bell's test experiment.

   (a) From the data compute the value of the CHSH quantity

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b').$$

   Does the data violate the bound on local hidden variable theories?

   (b) Assuming the photons were prepared ideally, in the state $\psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$, what does quantum mechanics predict for the quantity S given the polarizer settings a, b, a', b' used in the experiment?

   If you need a refresher, the posted paper by Dehlinger, et al., explains how to compute the expection values $E(\cdot,\cdot)$ from the raw photon counts.

6. Suppose that, in our Bell test experiment, we had failed to create an entangled state of HH and VV photons, but rather created a mixture consisting half of the state $|HH\rangle$ and half the state $|VV\rangle$.

   (a) Compute the expectation value $E(a, b)$ in this mixed state. There are two ways to get the answer: Either tabulate probabilities (easier) or use the density matrix (more challenging). Bonus points will be given for a correct treatment via the density matrix.

   (b) Using your formula for E, compute the CHSH quantity S for the detector settings used in our experiment. Would we have seen a violation of the bound on local hidden variable theories?