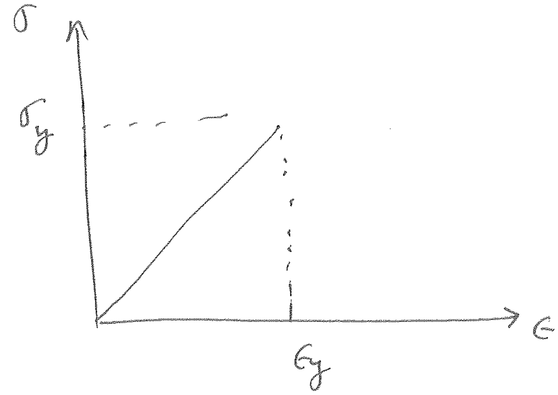
Mechanical Properties

1/ (a)



(b) Fe has the smallest Y of the given choices. Therefore, it will extend/compress the most.

(c) The slopes will differ. Y is effectively the slope of $\sigma(\epsilon)$. The actual values of ϵ_y (and of course σ_y) will differ. They're all linear, with a yield point.

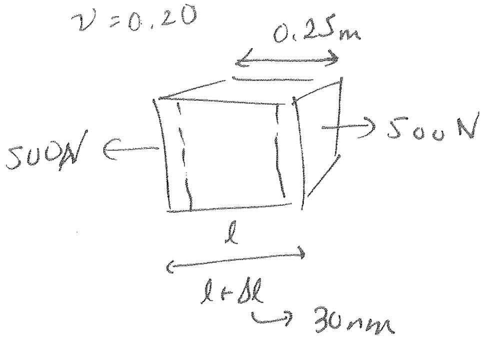
2/ (a) An intensive property is a property dependent on the type of material. For example, is it Al or Fe?

(b) An extensive property, is one that depends not only on the material (what is it), but also it's geometry (what's its shape?) or amount (how much of it is there?).

(c)

Intensive	Extensive
resistivity, ρ	resistance, R
mass density, ρ	mass, M
conductivity, σ	conductance, G
specific heat	heat capacity

3/ $\nu = 0.20$



(a) $\sigma = \dfrac{F}{A} = \dfrac{500\,N}{(0.25\,m)^2} = 8\ kPa$

(b) $\epsilon = \dfrac{\Delta l}{l} = \dfrac{30 \cdot 10^{-9}}{0.25} = 1.2 \cdot 10^{-7}$

(c) $Y = \dfrac{\sigma}{\epsilon} = \dfrac{8\ kPa}{1.2 \cdot 10^{-7}} = 67\ GPa$

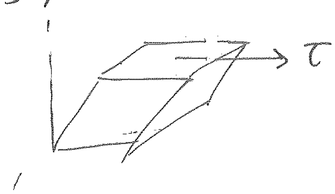
(d) $\dfrac{\Delta l_e}{l} = -\nu \dfrac{\Delta l_1}{l_1}$

$= -0.2\,(1.2 \cdot 10^{-7}) = -0.24 \cdot 10^{-7}$

so $\Delta l_e = (-0.24 \cdot 10^{-7})\,(0.25) = -6 \cdot 10^{-9}\,m$

the (−) indicates the lateral/side lengths decrease

4/ $G = 45\ GPa$



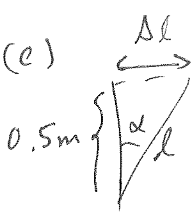
(a) $\tau = \dfrac{F}{A} = \dfrac{2250}{(0.5)^2} = 9\ kPa$

(b) $G = \dfrac{\tau}{\alpha}$

$\alpha = \dfrac{\tau}{G} = \dfrac{9 \cdot 10^{3}}{45 \cdot 10^{+9}} = 2 \cdot 10^{-7}$ rad.

in degrees $2 \cdot 10^{-7}\,\text{rad}\ \dfrac{360\,\text{deg}}{2\pi\,\text{rad}} = 11 \cdot 10^{-6}$ deg

(c)



small angles..

$\alpha \simeq \dfrac{\Delta l}{hyp.}$

in radians

$\Delta l = \alpha \cdot hyp = (2 \cdot 10^{-7})(0.5) = 0.1\,\mu m$

5/ $K = \frac{-p}{\Delta V / V}$ $\quad p = 6.15 \cdot 10^9$ Pa

$V = \frac{4}{3}\pi r^3$

ΔV? $\quad \Delta V \approx dV \quad d(V) = d(\frac{4}{3}\pi r^3)$

$= \frac{4}{3}\pi (3r^2) dr$

$dV = 4\pi r^2 dr$

$\Delta V = 4\pi r^2 \Delta r$

$$K = \frac{-p}{\frac{4\cancel{\pi} \cancel{r^2} \Delta r}{\frac{4}{3}\cancel{\pi} r^{\cancel{3}}}} = -\frac{p r}{3\Delta r} = \frac{(6.15 \text{ GPa})\left(\frac{1}{0.01}\right)}{3} \quad \text{when } \frac{\Delta r}{r} = 1\% = 0.01$$

$= 205$ GPa

6/ (a) $V + \Delta V = (l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3)$

$= l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 . l_3 + l_1 l_2 \Delta l_3$

$+ l_1 \Delta l_2 \Delta l_3 + \Delta l_1 l_2 \Delta l_3 + \Delta l_1 \Delta l_2 l_3$

$\Delta l_1 \Delta l_2 \Delta l_3$

keeping only terms w/ one Δl term or less

$V + \Delta V \approx l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3$

(b) if Δl is small ($\ll 1$), then the product $\Delta l \Delta l$ is a power of 2 smaller. That is $l \Delta l \gg \Delta l \Delta l$

(c) $\Delta V = (1 - 2\nu)(\Delta l_1 l_2 l_3)$

at $\nu = 0.5 \quad \Delta V = 0$

if $\nu > 0.5$ then $\Delta V < 0$ for $\Delta l > 0$ this isn't physically realistic.

that's saying an extension $\Delta l > 0$ leads to a smaller volume $\Delta V < 0$.

6 (d) $G = \dfrac{Y}{2(1+\nu)}$

- case that $\nu = -1$ $G \to \infty$

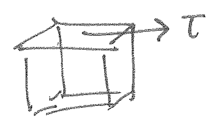
  since $G = \dfrac{\tau}{\alpha}$, $G \to \infty$ is the case of **no** deformation ($\alpha \to 0$)

  this is ok.

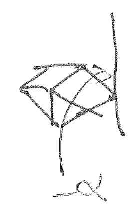
- however, if $\nu < -1$, say $\nu = -2$

  since $G = \dfrac{\tau}{\alpha}$, means that $\alpha < 0$ for $\tau > 0$

that is it deforms in direction *opposite* the shear force.



leads to

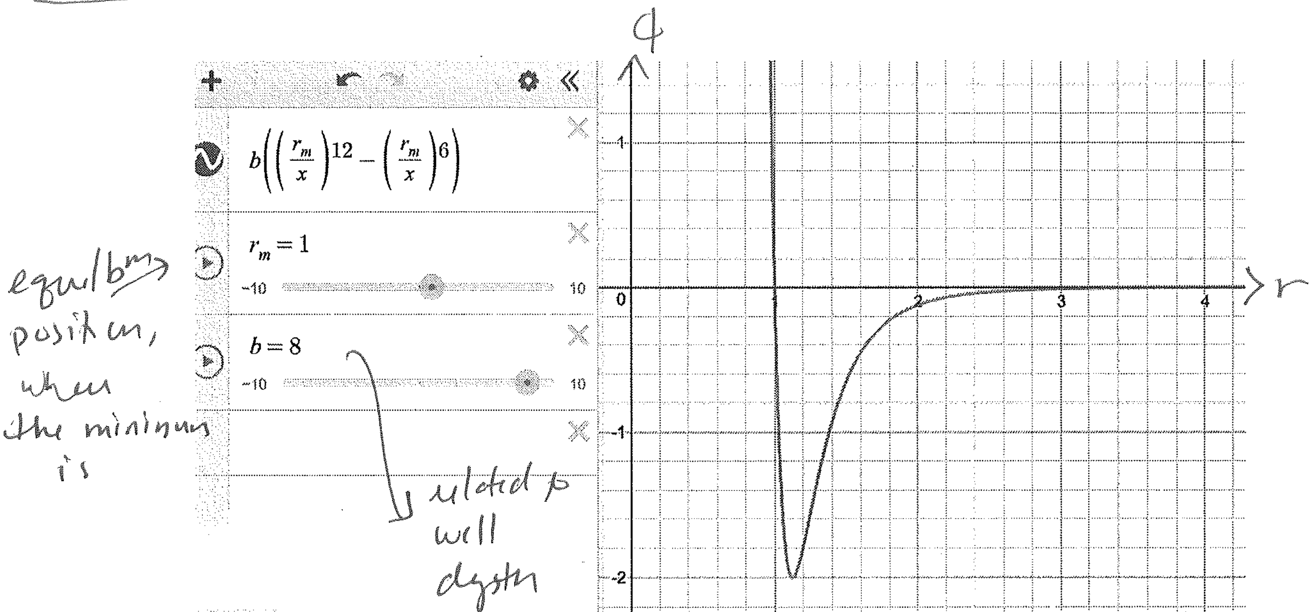


that's not physically realistic

7/ (a) try desmos.org, if you haven't already. See graph below

(b) Taylor series $f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots \frac{f^n(x_0)}{n!}(x-x_0)^n$

here, x_0 is the position around which you want to expand. In our case, it's the minimum position, r_m



graph for (a)

$$\phi(r_m) = b(1-2) = -b$$

$$\phi'(r_m) = \left.\frac{d\phi}{dr}\right|_{r_m} = b\left[-12\, r_m^{12}\, r^{-13} - 2(-6)\, r_m^{6}\, r^{-7}\right]\Big|_{r_m}$$

$$= b\left[-12\, r_m^{-1} + 12\, r_m^{-1}\right] = 0$$

$$\phi''(r_m) = \left.\frac{d^2\phi}{dr^2}\right|_{r_m} = b\left[(-12)(-13)\, r_m^{12}\, r^{-14} - 2(-6)(-7)\, r_m^{6}\, r^{-8}\right]\Big|_{r_m}$$

$$= b\left[156\, r_m^{-2} - 84\, r_m^{-2}\right] = \frac{72b}{r_m^2}$$

7 cont'd/ so $\phi(r) = -b + \frac{36b}{r_m^2}(r-r_m)^2$ + higher order terms

(c) for small displacements, the "higher order terms" are negligible compared to the first 2 terms.

Therefore, $\phi(r)$ has the same form as the SHO $U = \frac{1}{2}kr^2 + U_0$

Note that offsets like U_0, $-b$, don't affect the behavior of systems; our theory should not depend on our reference point.

8/ (a) Sine is a periodic func - it reflects the periodicity of the lattice. In addition, sine starts at ϕ.

$\tau \to 0$ when $\alpha \to 0$.

In this case, this models the behavior that small shear stress is associated with small shear angles

(b) sin repeats every 2π.

Therefore, $x = 0$ is for $\theta = \frac{2\pi \cdot 0}{b} = 0$

$x = b$ is for $\theta = \frac{2\pi b}{b} = 2\pi$

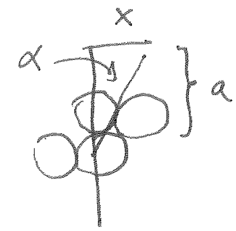
periodicity is b

max value of sin $= 1$. Therefore max value of $\tau = C$

9/ (a) macroscopically,

$G = \frac{\tau}{\alpha}$

this same angular displacement on the microscopic scale is



here, $\tan \alpha = \frac{x}{a}$ in the limit α is small, use the small angle approx, $\tan\alpha \approx \alpha$

therefore, $\alpha = \frac{x}{a}$

$G \approx \frac{\tau}{x/a}$

$\tau \approx Gx/a$

(b) from (a) $\tau \approx Gx/a$

from (8) $\tau = C \sin\left(\frac{2\pi x}{b}\right)$

$\approx C\left(\frac{2\pi x}{b}\right)$ again, small angles... $\sin\theta \approx \theta$

and so $\frac{Gx}{a} = C\left(\frac{2\pi x}{b}\right)$

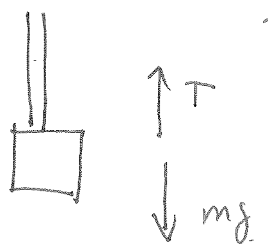
$C = \frac{Gb}{2\pi a}$

(c) b and a are lattice constants... distances between lattice sites in 2 directions. Though $b \neq a$, often; they're of the same order of magnitude. C is τ_y, the yield stress

(d) $\tau_y = G \frac{b}{2\pi a}$

10/ The observed yield stress is <u>less</u>. Dislocations in the crystal allow motion to occur with less energy. Instead of having to move a whole plane of atoms at once, only individual rows have to move at any given time.

11/



$$\sum F = ma$$

$$T - mg = 0$$

$$T = mg$$

$$\sigma_y \approx (0.2)(0.2\ GPa)$$

$$= 40\ MPa$$

$$\sigma_y = \frac{T_{max}}{Area}$$

$$m = \frac{T}{g}$$

$$= \frac{\sigma_y\, Area}{g}$$

$$= \frac{(40 \cdot 10^6)(\pi)(1 \cdot 10^{-2})^2}{9.8}$$

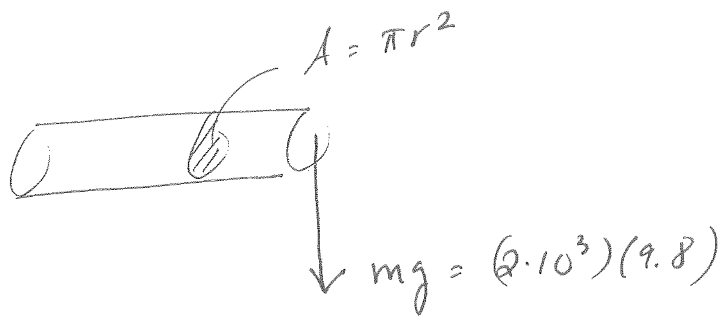
$$m = 1282\ kg$$

$$m = m_{elev} + (N_{people} \times 70)$$

$$\frac{m - m_{elev}}{70} = N_{people}$$

$$\frac{1282 - 150}{70} = 16.2 \text{ people}$$

12/



(a) $\tau = \dfrac{\text{Force}}{\text{area}} = \dfrac{mg}{\pi r^2} = \dfrac{(2\cdot10^3)(9.8)}{\pi(1.5\cdot10^{-2})^2} = 2.7\cdot10^7\ \text{Pa} = 27\ \text{MPa}$

(b) $G = \dfrac{\tau}{\alpha}$

$\alpha = \dfrac{\tau}{G} = \dfrac{27\cdot10^6}{84\cdot10^9} = 3\cdot10^{-4}\ \text{rad}$ (or 0.018 degrees)

13/ (a) glass cables are brittle .. leading to immediate fracture & therefore failure right at the yield stress.
Steel, on the other hand, has a plastic deformation region, at the yield point. leading to deformation, but not failure,

(b)