Mechanical properties

1. (a) Sketch a typical stress-strain curve in the elastic deformation region..

   (b) Consider the following materials: diamond (950 GPa), iron (Fe, Young's modulus of 120 GPa), and tungsten (W, 350 GPa). Which one extends or compresses the most when subject to a tensile or compressive force?

   (c) For these materials, how do the strain-stress curves in the elastic region differ? How are they similar?

2. Young's modulus describes an intensive property.

   (a) What's the difference between an intensive property and an extensive property? Which one is sample-dependent (i.e. depends on the specific geometry or amount of matter), and which is only material-dependent?

   (b) Identify each of the following as an intensive or extensive property

   resistance, resistivity, mass density, mass, conductance, conductivity, heat capacity, specific heat

3. A cube (Poisson's ratio of 0.20 and side length of 0.25m) has 500N of normal force pulling outwards on opposite faces. The two faces extend apart by 30nm. Calculate

   (a) the normal stress
   (b) the normal strain
   (c) Young's modulus
   (d) the lateral displacement of the sides of the block that don't have a normal force applied to them. Do the side lengths increase or decrease?

   from insula.com.au. 8kPa, $0.12 \cdot 10^{-6}$*, 67 GPa, -6.3nm*

4. A copper cube with side length 0.50m is subjected to a shear force of 2250N along its top face. Calculate the

   (a) shear stress
   (c) the angle of shear
   (d) the linear displacement of the top face.

   Use a modulus of rigidity of 45 GPa.

   from insula.com.au. 9kPa, $0.2 \cdot 10^{-6}$, $12 \cdot 10^{-6}$ *degrees, 1μm*

5. A solid sphere is immersed in a liquid so that the hydrostatic pressure is 6.15 GPa. The sphere contracts such that its diameter decreases by 1%. Calculate the bulk modulus of the sphere.

   from insula.com.au. 205 GPa

6. Poisson's ratio can have a range of $-1 < \nu < 0.5$. Let's look at the upper and lower limit for ν.

   (a) The volume of a solid with a stress applied is

   $$V + \Delta V = (\ell_1 + \Delta\ell_1)(\ell_2 + \Delta\ell_2)(\ell_3 + \Delta\ell_3)$$

   Expand this product. Keep only terms with none or one $\Delta\ell$ term. You should have 4 terms left.

   (b) In the above, how do we justify dropping terms such as $\ell_1\Delta\ell_2\Delta\ell3$?

   (c) From (a) we have $\Delta V = \Delta\ell_1\ell_2\ell_3 + \ell_1\Delta\ell_2\ell_3 + \ell_1\ell_2\Delta\ell_3$. If we substitute in $\frac{\Delta\ell_2}{\ell_2} = \frac{\Delta\ell_3}{\ell_3} = -\nu\frac{\Delta\ell_1}{\ell_1}$, we find that

   $$\Delta V = (1 - 2v)\Delta\ell_1\ell_2\ell_3$$

   Use the above to argue that ν has an upper limit of 0.5.

   (d) The modulus of rigidity and Young's modulus are related by

   $$G = \frac{Y}{2(1+\nu)}$$

   Use the above to argue that ν has a lower limit of -1.

7. The Lennard-Jones potential is a simple model of the interaction between two neutral molecules.

   $$\phi_{\text{LJ}} = b\left[\left(\frac{r_m}{r}\right)^{12} - 2\left(\frac{r_m}{r}\right)^6\right]$$

   It has a minimum at $r = r_m$, with a depth of b.

   (a) Sketch this potential. You're welcome to use any graphing software/app.
   (b) Do a Taylor series expansion of this potential around $r = r_m$. Provide the first 2 non-zero terms.
   (c) Use the above to show that, at small displacements from the equilibrium position, this potential is that of a simple harmonic oscillator (that is, a Hooke's law spring).

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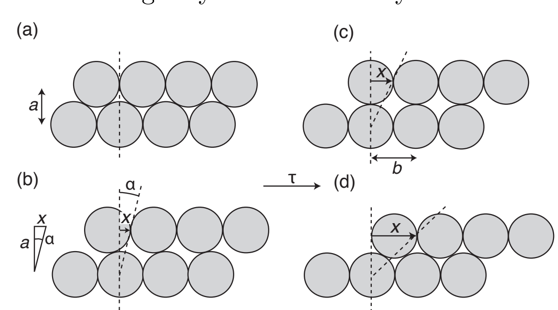
8. In the simplest model of yield stress for shearing a solid, the shear stress depends on the displacement x where

   $$\tau = C\sin\left(\frac{2\pi x}{b}\right)$$

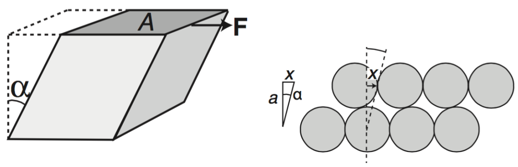
   (a) Explain the reason for using a sine function. The following may be useful. Use your own words.

   (b) Use the above expression for τ determine its (spatial) period and maximum value.

9. Here, we find an estimate for the shear stress at the yield point, τ_γ.

   Macroscopically, we describe the shear stress as $G = \tau/\alpha$. Macroscopic motion is result of microscopic motion between layers.



(a) Show that

$$\tau \approx G\frac{x}{a}$$

Use the above picture, the definition of G, and the small angle approximation.

(b) Use your result from (a) and problem 8 to show that

$$C = G\frac{b}{2\pi a}$$

(c) Here, we approximate $b \approx a$. What do a and b represent and why is this a reasonable approximation? What does C represent (see Prob 8)?

(d) Write the final expression for this estimate

$$\tau_\gamma \approx \ldots$$

This line of reasoning is given in both Turton eq 3.11 and ex3.1, and Hofmann p39-40

10. The yield stress of a solid estimated from a simple calculation is often much higher than the observed yield stress. Why?

    Hofmann 3.q5

11. A lift cable of diameter 2cm has a yield stress of 0.2GPa. An empty lift has a mass of 150kg, and the average mass of a person is 70kg. Given that a safe operating limit is 20% of the elastic limit, determine the maximum number of people that can be in the lift.

    Turton 3.5. 16 people

12. A steel bar of length 2m and diameter 3cm projects horizontally from a building. The elastic limit is reached when a mass of 2000kg is suspended from the free end. Calculate the

    (a) yield stress
    (b) angle of shear at the elastic limit

    Turton 3.6. 27.8 MPa, 3.3 x 10^{-4} rad

13. (a) The yield stress of glass is considerably higher than the yield stress of steel (600 MPa for glass, 200 MPa for steel). Why aren't glass cables used instead of steel cables in load bearing applications?

    (b) Sketch representative stress-strain curves for glass and steel.

    Turton 3.8 and Hofmann 3.q3