$\boxed{\text{Thermal properties, part II}}$

1/ (a) there are 3 independent terms here

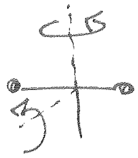
$$\langle E \rangle = \left(3 \times \tfrac{1}{2} k_B T\right) \times N_A$$

$$\langle E \rangle = \frac{3}{2} N_a k_B T$$

(b)

$$C = \frac{\partial \langle E \rangle}{\partial T} = \frac{3}{2} N_a k_B$$

(c), (d)



$$E = N_a\left(\tfrac{1}{2} m v_x^2 + \tfrac{1}{2} m v_y^2 + \tfrac{1}{2} m v_z^2 + \tfrac{1}{2} I_1 \omega_1^2 + \tfrac{1}{2} I_2 \omega_2^2\right)$$

$$\langle E \rangle = N_a \times \left(5 \times \tfrac{1}{2} k_B T\right) = \frac{5}{2} N_a k_B T$$

$$C = \frac{\partial \langle E \rangle}{\partial T} = \frac{5}{2} N_a k_B$$

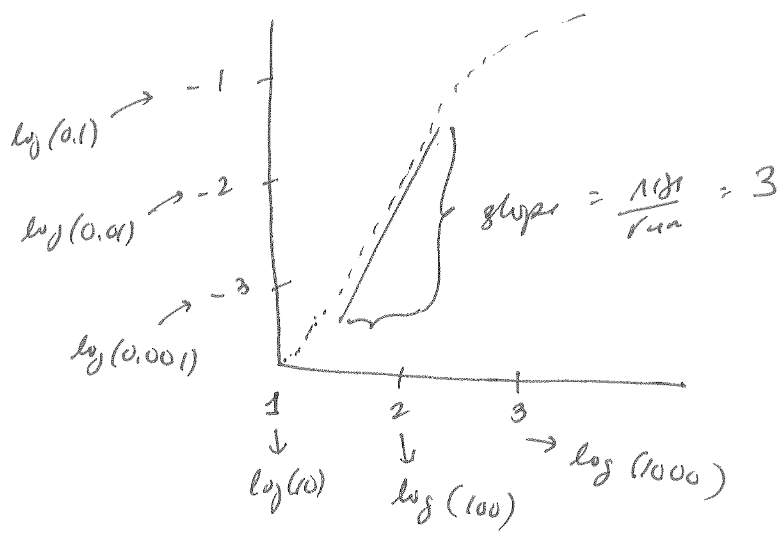
~~(e)~~ (f)

$$E = N_a\left(\tfrac{1}{2} m v_x^2 + \tfrac{1}{2} m v_y^2 + \tfrac{1}{2} m v_z^2 + \tfrac{1}{2} I_1 \omega_1^2 + \tfrac{1}{2} I_2 \omega_2^2 + \tfrac{1}{2} m (\Delta \dot{x})^2 + \tfrac{1}{2} k (\Delta x)^2\right)$$

$$\langle E \rangle = N_a \times 7 \times \tfrac{1}{2} k_B T = \frac{7}{2} N_a k_B T$$

$$C = \frac{\partial \langle E \rangle}{\partial T} = \frac{7}{2} N_a k_B$$

2/ slope = 3



$$\frac{\log c}{\log T} = 3$$

$$\log c = 3 \log T$$

$$C \propto T^3$$

3/ $C = \frac{\partial \langle E \rangle}{\partial T}$

$$= \frac{\partial}{\partial T}\left[\frac{1}{2}\hbar\omega_E + \frac{\hbar\omega_E}{e^{\hbar\omega_E/k_B T} - 1}\right]$$

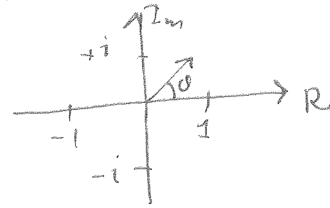
$$= \hbar\omega_E (-1)\left(e^{\hbar\omega_E/k_B T} - 1\right)^{-2}\left(\frac{\hbar\omega_E}{k_B}(-1)T^{-2} e^{\hbar\omega_E/k_B T}\right)$$

$$c = \frac{\left(\frac{\hbar\omega_E}{k_B T}\right)^2 e^{\hbar\omega_E/k_B T}}{\left(e^{\hbar\omega_E/k_B T} - 1\right)^2}$$

* differs from 4.32
since $R = N_a k_B$
factor $3N_a$ not included in our
$\langle E \rangle$.

4/ (a) $e^{i[kx-\omega t]} = e^{i[k(x+L)-\omega t]}$

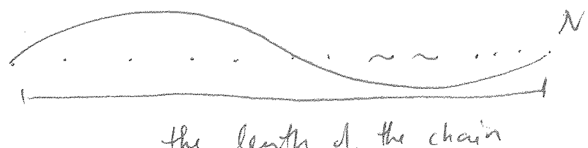
$1 = e^{ikL}$ in the Re-Im plane $e^{i\theta} = 1$ when $\theta = 0, 2\pi, 4\pi, \dots$



$kL = 2p\pi$ when $p = 1, 2, 3 \dots$

$k = \frac{2\pi p}{L}$

(b) $k = \frac{2\pi}{L}$ since $k = \frac{2\pi}{\lambda}$, $\lambda = L$



long wavelength limit

(c) $k = \frac{2\pi N}{L}$ since $k = \frac{2\pi}{\lambda}$, $\lambda = \frac{L}{N} = \frac{Na}{N} = a$

short wavelength

5/ (a) $\Delta k = k_f - k_i$

$= \frac{10.03\pi}{L} - \frac{2.03\pi}{L} = \frac{8\pi}{L}$

(b) 4 states

(c) $g(k) = \frac{dN}{dk} = \frac{4}{8\pi/L} = \frac{L}{2\pi}$

(d) $dN = g(k)\,dk$

$= \left(\frac{L}{2\pi}\right)\left(\frac{76\pi}{L}\right) = 38$ states

6/(a) Circle has area $A = \pi k^2$

# of states in this area: $N = \pi \left(\frac{k}{2\pi/L}\right)^2$

$$N = \frac{k^2 L^2}{4\pi}$$

(b) $g(k) = \frac{dN}{dk} = \frac{2kL^2}{4\pi} = \frac{kL^2}{2\pi}$

(c) $N(k) = \frac{k^2 L^2}{4\pi}$ $\quad\quad w = vk$

$k = \frac{w}{v}$

$$N(w) = \left(\frac{w}{v}\right)^2 \frac{L^2}{4\pi}$$

$$N(w) = \frac{w^2 L^2}{4\pi v^2}$$

(d) $g(w) = \frac{dN}{dw} = \frac{2wL^2}{4\pi v^2} = \frac{wL^2}{2\pi v^2}$

7/ $$3N = 3\int_0^{w_D} g(w)\,dw$$

$$N = \int_0^{w_D} \frac{w^2 V}{2\pi^2 v^3}\,dw = \frac{V}{2\pi^2 v^3}\int_0^{w_D} w^2\,dw$$

$$N = \frac{V}{2\pi^2 v^3}\,\frac{w_D^3}{3}$$

$$w_D^3 = \frac{6\pi^2 v^3 N}{V}$$

8/(a) $\langle E \rangle = 3\int_0^\infty \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} g(\omega)\, d\omega$

here, $g(\omega) = \frac{\omega^2 V}{2\pi^2 v^3}$ and ω has the range $0 \to \omega_D$

$$\langle E \rangle = 3\int_0^{\omega_D} \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} \frac{\omega^2 V}{2\pi^2 v^3} d\omega$$

$$\langle E \rangle = \frac{3\hbar V}{2\pi^2 v^3} \int_0^{\omega_D} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1} d\omega$$

(b) let $x = \frac{\hbar\omega}{k_B T}$ and $dx = \frac{\hbar\, d\omega}{k_B T}$

$$\langle E \rangle = \frac{3\hbar V}{2\pi^2 v^3} \int_0^{x_D} \frac{\left(\frac{k_B T}{\hbar} x\right)^3 \frac{k_B T\, dx}{\hbar}}{e^x - 1}$$

$$= \frac{3\hbar V}{2\pi^2 v^3} \left(\frac{k_B T}{\hbar}\right)^4 \int_0^{x_D} \frac{x^3}{e^x - 1} dx$$

$$= \underbrace{9 N k_B T \left(\frac{T}{\Theta_D}\right)^3}_{} \int_0^{x_D} \frac{x^3}{e^x - 1} dx$$

check to see

$9 N k_B T \left(\frac{T}{\frac{\hbar\omega_D}{k_B}}\right)^3$, when $\omega_D^3 = \frac{6\pi^2 N v^3}{V}$

$9 N k_B T^4 \left[\frac{k_B^3}{\hbar^3 \left(\frac{6\pi^2 N v^3}{V}\right)}\right]$

$\frac{3\hbar}{2\hbar^3} \frac{V \cdot k_B^4 T^4}{\pi^2 v^3}$ ✓

9/

(a) $e^x \approx 1 + x$

(b) $\langle E \rangle = 9 N k_B T \left(\frac{T^3}{\Theta_D}\right)^3 \int_0^{x_D} \frac{x^3}{e^x - 1}\, dx$ when $e^x \approx 1 + x$

$= \quad . \; . \; . \; . \; . \quad \int_0^{x_D} \frac{x^3}{1 + x - 1}\, dx$

$. \; . \; . \; . \quad \int_0^{x_D} x^2\, dx$

$= \quad . \; . \; . \; . \quad \frac{x_D^3}{3}$ when $x_D = \frac{\Theta_D}{T}$

$= 9 N k_B T \left(\frac{T}{\Theta_D}\right)^3 \frac{1}{3} \left(\frac{\Theta_D}{T}\right)^3$

$\langle E \rangle = 3 N k_B T$

(c) $C = \frac{\partial \langle E \rangle}{\partial T} = 3 N k_B$

10/

(a) $\int_0^{\infty} \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15} = 6.494$

(b) $\langle E \rangle = 9 N k_B \frac{T^4}{\Theta_D^3} (6.494)$

(c) $C = \frac{\partial \langle E \rangle}{\partial T} = \frac{234\, N k_B T^3}{\Theta_D^3}$ note T^3 dependence.

11/ (a) $K = \frac{1}{3} c \lambda_p v_p$

$$\lambda_p = \frac{3K}{C v_p}$$

$$= 3 \underbrace{\left(2200 \frac{W}{mK}\right)}_{K} \underbrace{\left(\frac{mol\ K}{5\ J}\right)}_{c} \underbrace{\left(\frac{s}{5000\ m}\right)}_{v_p} \left(\frac{12 \cdot 10^{-3}\ kg}{1\ mol} \frac{m^3}{3.52\ kg}\right)$$

need the heat capacity of the solid per unit volume

$$= 8 \cdot 10^{-7} m$$

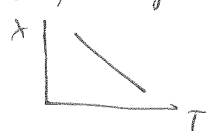
$$= 0.8\ \mu m$$

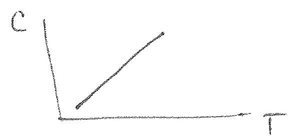
(b) average distance a phonon travels before colliding w/ another phonon, lattice imperfection

12/ $K(T) = \frac{1}{3} c\, \lambda_p v_p$

$C(T)$ increases w/ Temperature

$\lambda_p(T)$ decreases w/ temperature





the product of these 2 will lead to a peak in K

13/ (a)

diamond	~2000
copper	~400
stainless steel	12 – 45
glass	0.8
concrete	0.1 – 1.7
asbestos	0.08
styrofoam (polystyrene)	0.03

(b) $$\frac{dQ}{dt} = K_1 A \frac{dT}{dx_1} = K_2 A \frac{dT}{dx_2}$$

$$\frac{K_1}{dx_1} = \frac{K_2}{dx_2}$$

$$dx_2 = \frac{K_2}{K_1} dx_1$$

avg of 0.9

$$= \frac{0.9}{0.03} (2\ in)$$

$$dx_2 = 60\ in$$

14/

$$2d_1 \sin\theta_1 = m\lambda$$

$$2d_2 \sin\theta_2 = m\lambda$$

$$\underbrace{d_1 \sin\theta_1}_{\text{at } 300K} = \underbrace{d_2 \sin\theta_2}_{\text{at } 500K}$$

expansion $\frac{\Delta d}{d} = \alpha \Delta T$

$$\frac{d_2 - d_1}{d_1} = \frac{\frac{d_1 \sin\theta_1}{\sin\theta_2} - d_1}{d_1} = \frac{\sin\theta_1}{\sin\theta_2} - 1 = \alpha \Delta T$$

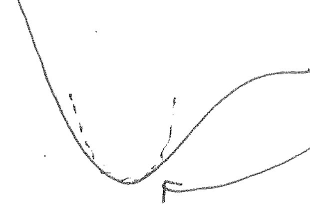
$$\alpha = \frac{\frac{\sin\theta_1}{\sin\theta_2} - 1}{\Delta T} = \frac{\frac{\sin(25.23)}{\sin(25.14)} - 1}{200} = 16 \cdot 10^{-6}$$

15/ We often expand $U(r)$ in terms of a Taylor series around its minimum

(a)

$$U(r) \cong r_0 + \underbrace{\cancel{U'(r_0)(r-r_0)}}_{=0} + \underbrace{\tfrac{1}{2}U''(r_0)(r-r_0)^2}$$

harmonic term describes the harmonic region where it is approx. quadratic + symmetric about r_0.



the higher order terms $\frac{1}{3!}U'''(r_0)(r-r_0)^3 + \dots$ are the anharmonic terms.

(b) See your text