Electrical properties of metals - classical approach

1. (a) Derive the following expression for average the average thermal velocity of electrons in a metal.

$$v_t = \sqrt{3k_bT/m}$$

   Calculate the thermal speed of electrons at
   (b) room temperature (70^{o}F)
   (c) water's freezing point (32^{o}F)

2. (a) What meant by *relaxation time* and *mean free path* for electrons in a metal? Use words and a diagram.

   (b) From (1b) and a mean free path of 1nm[1], give an estimate of the relaxation time.

3. (a) Derive the following expression for the drift velocity

$$\bar{v} = \frac{-eE\tau}{m}$$

   Start with Newton's second law. Propose a $v(t)$ to solve the d.e. Clearly state any assumptions made in the derivation.

   (b) Calculate the electron drift velocity associated with an electric field of 100 V/m (e.g. a 9V difference over 9cm) and a relaxation time of 10^{-14}s.

4. The microscopic form of Ohm's law is $J = \sigma E$. From this, derive the macroscopic form

$$\Delta V = IR$$

   Use the definition of current density ($J = I/A$) and the relationship between electric field and potential difference ($E = \Delta V/d$ for a uniform electric field). What must constants make up R?

5. (a) Lithium has a density of 530 kg/m^3, molar mass of 7 g/mol, and is in Column I of the periodic table. Calculate the number density of valence electrons in lithium (electrons/m^3).

   (b) Assume that the relaxation time in lithium is (8.6×10^{-15})s at 295K. Calculate lithium's conductivity.

   ($4.6 \times 10^{28}/\mathrm{m}^3, 1.1 \times 10^7\Omega^{-1}\mathrm{m}^{-1}$)

6. (a) Aluminum has a density of 2700 kg/m^3, molar mass of 27 g/mol, and is in Column III of the periodic table. Calculate the number density of valence electrons.

   (b) Aluminum has a resistivity of 2.5 ×10^{-8}Ω·m at room temperature. Calculate the relaxation time in aluminum.

   ($18.1 \times 10^{28}/\mathrm{m}^3$, 8×10^{-15} s)

7. What is the Drude model for electrical conductivity in metals? In particular

   (a) Describe the motion of electrons in a metal when there's no electric field. Use the following terms in your description: thermal velocity, mean free path, relaxation time.

   (b) Describe the motion of electrons in a metal, when there is an electric field. Use the following terms: drift velocity

   (c) How fast do electrons move in the Drude model, when there's no electric field? When an electric field is applied, how does the additional speed of the electrons compare to their thermal speed?

   (d) Where does electrical resistance come from in the Drude model?

8. List two successes of the Drude model. How is this successful?

9. The Drude model has some shortcomings.

   (a) It predicts a wrong temperature dependence for the resistivity. Experimentally, $\rho(T) \sim T$. Show that the Drude model gives the following dependence for resistivity

$$\rho(T) \sim T^{\frac{1}{2}}$$

   Start with the thermal velocity and how it depends on temperature.

   (b) Give another shortcoming or failure of the Drude model. You'll have to read the text. Write 1-2 sentences to describe the problem. Include numerical values or equations to quantify your answer.

[1] we're estimating that electrons can go a few lattice spacings before colliding with an ion