Semiconductors

1. At $T = 0$, there are no electrons in a semiconductor's conduction band. However, for $T > 0$, thermal excitations lead to a finite number of conduction electrons. Here, we determine an expression for the density of electrons, $n = N/V$, in the conduction band.

   The number of electrons at between energy E and $E + \Delta E$ is given by

   $$N = \int_{E}^{E+\Delta E} g(E) f(E,T) dE$$

   (a) Name each of the terms the integral. What do they physically represent?

   (b) At 300K, $f(E,T)$ can be approximated as

   $$f(E,T) \approx e^{(E_f - E)/k_b T}$$

   Show how this approximation comes about.

   The density of states in the conduction band can be approximated as

   $$g(E) = \frac{V}{2\pi^2} \frac{2m}{\hbar^2}^{3/2} (E - E_g)^{1/2}$$

   This is the free-electron d.o.s., shifted so that the top of the valence band is $E = 0$. Substituting in $g(E)$ and $f(E,T)$ leads to

   $$n = \frac{N}{V} = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_{E_g}^{\infty} (E - E_g)^{1/2} e^{(E_f - E)/k_b T} dE$$

   This integral can be evaluated using the change of variable $x = (E - E_g)/k_b T$, and becomes

   $$n = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} e^{(E_f - E_g)/k_b T} \int_{0}^{\infty} x^{1/2} e^{-x} dx$$

   (c) Evaluate the integral. Show that it leads to

   $$n = 2 \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_f - E_g)/k_b T} \qquad (1)$$

2. Using the result from problem 1, calculate the density of conduction electrons for Si at

   (a) 300K

   (b) 302K

   For Si, the energy gap is 1.11eV, and the fermi energy is $E_f = E_g/2$ (we're setting the top of the valence band to be $E = 0$).

   (answers on the order of 10^{15} electrons/m^3)

3. Determine the lowest frequency photon required to excite an electron into the conduction band for

   (a) Si, with a band gap of 1.11eV

   (b) diamond, with a band gap of 5.5eV

   (c) What parts of the EM spectrum do these correspond to? Why is Si opaque and diamond transparent?

4. What's a *hole*?

5. One of the ways to determine the effective mass m^* of charge carriers in a material is to measure the cyclotron frequency ν_c. In this measurement, charge carriers move in a circular orbit when placed in an external magnetic field B. The charge can absorb only photons whose frequency matches the frequency of the circular orbit, the cyclotron frequency ν_c.

   (a) Derive the expression

   $$\nu_c = \frac{eB}{2\pi m_e}$$

   Start with Newton's second law. Keep in mind that $a = v^2/R$ for uniform circular motion, the force from B is the standard qvB, and period and frequency are related $T = 1/\nu$.

   (b) Calculate the cyclotron frequency for a free electron in a 0.05T magnetic field.

   (c) In GaAs, the cyclotron frequency for an electron is 21.5 GHz in a 0.05T field. Determine the effective mass of the electron m_e^*. Give this in terms of m_e.

   1.4 GHz, $0.065 m_e$

6. What is meant by *effective mass*, as thought of in semiconductor systems?

7. Semiconductors can be doped to create a material with a conductivity most appropriate to your application.

   (a) Determine the concentration of conduction electrons in a sample of silicon if one in every million Si atoms is replaced by a phosphorus atom.

   Assume that every phosphorus atom is single ionized. Si has a molar mass of 0.028 kg/mol and a density of 2300 kg/m^3.

   (b) It turns out that doping determines the conductivity, more than material's intrinsic conductivity. Give a comparison or calculation that shows this.

   (4.9×10^{22}m^{-3})