P2.9

Starting with eq 2.21 for the field we have

$$\mathbb{E} = \mathbb{E}_0 e^{-\frac{2\pi\omega}{c}\hat{u}\cdot\mathbb{r}} \cos\left(\frac{n\omega}{c}\hat{u}\cdot\mathbb{r} - \omega t + \phi\right)$$

$\mathbb{E}_0$ is real. phase is here

with $x = 0$ we know that $d\mathcal{N} = n$

and so $\mathbb{k} = \frac{\omega}{c} n \hat{u}$ and

$$\mathbb{B}_0 = \frac{\mathbb{k} \times \mathbb{E}_0}{\omega} = \frac{n}{c}\hat{u} \times \mathbb{E}_0$$

So

$$\mathbb{E} = \mathbb{E}_0 \cos(\theta - \omega t) \quad \text{with} \quad \theta \equiv \frac{n\omega}{c}\hat{u}\cdot\mathbb{r} + \phi$$

$$\mathbb{B} = \frac{n}{c}\hat{u} \times \mathbb{E}_0 \cos(\theta - \omega t)$$

thus

$$\mathcal{S} = \frac{1}{\mu_0}\mathbb{E}\times\mathbb{B} = \frac{1}{\mu_0}\frac{n}{c}\mathbb{E}_0 \times (\hat{u}\times\mathbb{E}_0)\cos^2(\theta - \omega t)$$

$$= \frac{1}{\mu_0\epsilon_0}\frac{n}{c}\epsilon_0\left[(\mathbb{E}_0\cdot\mathbb{E}_0)\hat{u} - (\mathbb{E}_0\cdot\hat{u})\mathbb{E}_0\right]\cos^2(\theta - \omega t)$$

($\mathbb{E}_0\cdot\hat{u} = 0$)

$$= c^2\frac{n}{c}\epsilon_0 E_0^2 \hat{u}\cos^2(\theta - \omega t)$$

$$\mathcal{S} = cn\epsilon_0 E_0^2 \cos^2(\theta - \omega t)$$

$$I = \langle \mathcal{S} \rangle_t = cn\epsilon_0 E_0^2 \frac{1}{2}$$

Done

P2.10 The power is

$$P = \frac{\Delta E}{\Delta t} = \frac{100\,mJ}{25\times10^{-15}\,s} = 4\times10^{12}\,\frac{J}{s}$$

The intensity is power per area

$$I = \frac{P}{A} = \frac{4\times10^{12}\,W}{\pi(5\,\mu m)^2} = 5.1\times10^{22}\,\frac{W}{m^2}$$

(a) $I = 5.1\times10^{18}\,\frac{W}{cm^2}$

(b) $I = nc\,\frac{1}{2}\epsilon_0 E_0^2 = c\,\frac{1}{2}\epsilon_0 E_0^2$

$$\Rightarrow E_0 = \sqrt{\frac{2I}{c\,\epsilon_0}} = 6.19\times10^{12}\,\frac{V}{m} = 619\,\frac{V}{\text{Å}}$$

(c) $B_0 = \frac{n}{c}E_0 = \frac{E_0}{c} = 20600\,T$

P2.11 The radius of the image of the sun on the Retina will be

$$r_i = \frac{d_i}{d_o} r_{sun}$$

The power entering the eye is determined by the intensity of the light and the radius of the pupil

$$P = I_s \pi r_{pupil}^2$$

Assuming this power is uniformly distributed across the image we find the intensity on the retinal will be

$$I_R = \frac{P}{\pi r_i^2} = I_s \frac{r_{pupil}^2}{r_i^2} = I_s \left[\frac{r_{pupil}}{r_s} \frac{d_o}{d_i} \right]^2$$

(a) So $I_R = 133 \frac{kW}{m^2} = 13.3 \frac{W}{cm^2}$

(b) The size of the image of the beam is $r_i = \frac{d_i}{d_o} r_o = \frac{22\,mm}{2000\,mm} 0.5\,mm = 5.5\,\mu m$

The intensity on the retina is then

$$I_R = \frac{P}{A} = \frac{1.0\,mW}{\pi r_i^2} = 10\,\frac{MW}{m^2} = 1050\,\frac{W}{cm^2}$$

P2.12 We start by considering the motion of a free charge in an oscillating field, assuming that the electric force is the primary force.

$$ma = F = qE$$

$$\rightarrow a = \frac{q}{m} E_0 \cos(\omega t)$$

$$\rightarrow x = -\frac{1}{\omega^2} \frac{q}{m} E_0 \cos(\omega t)$$

$$\rightarrow v = \frac{1}{\omega} \frac{q}{m} E_0 \sin(\omega t)$$

Now compute the magnetic force give this motion.

$$F_B = q v \times B \Rightarrow F_B = q v B$$

and $$\frac{F_B}{F_E} = \frac{qvB}{qE} = \frac{B}{E} v$$

$$= \frac{k}{\omega} \frac{E}{E} v = \frac{k}{\omega} \frac{1}{\omega} \frac{q}{m} E_0 \sin \omega t$$

$$= \frac{1}{c^2} \frac{1}{k} \frac{q}{m} E_0 \sin \omega t$$

$$= \frac{\lambda}{2\pi c^2} \frac{q}{m} E_0 \sin \omega t$$

$$\max\left[\frac{F_B}{F_E}\right] = \frac{\lambda}{2\pi c^2} \frac{q}{m} E_0 = \frac{E_0}{E_{thresh}}$$

with $$E_{thresh} \equiv \frac{2\pi c^2}{\lambda} \frac{m}{q} = 3 \times 10^{12} \frac{N}{C}$$

Thus $$I_{thresh} = c \frac{1}{2} \epsilon_0 E_{thresh}^2 = 1.3 \times 10^{18} \frac{W}{cm^2}$$

Done

For fun general case

With actual Lorentz force

$$m\mathbf{a} = \mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$$

$$\mathbf{a} = \frac{q}{m}\left[E\hat{x} + \mathbf{v}\times\hat{y}B\right]$$

$$\frac{m}{q}\mathbf{a} = E\hat{x} + (v_x\hat{x} + v_y\hat{y} + v_z\hat{z})\times\hat{y}B$$

$$= E\hat{x} + (v_x\hat{z} - v_z\hat{x})B$$

$$= (E - v_zB)\hat{x} + v_xB\hat{z}$$

So

$$\frac{dv_x}{dt} = \frac{q}{m}(E - v_zB)$$

$$\frac{dv_y}{dt} = 0 \quad \Rightarrow \text{assume } v_y = 0 \text{ ; } y = 0$$

$$\frac{dv_z}{dt} = \frac{q}{m}v_xB$$