Exam 2 - solutions

1/ $G = 0.25 e^{+j\pi/4}$ this is of the form $G = |G| e^{j\Delta\phi}$

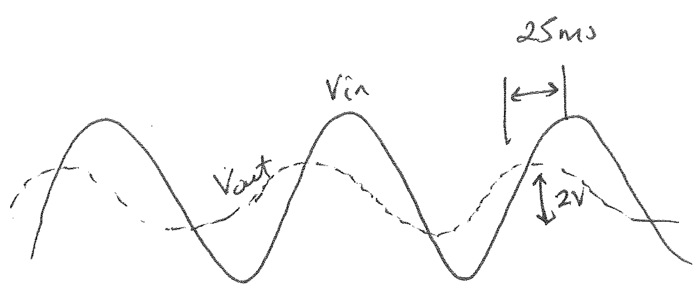
so $|G| = 0.25$

or $V_{out} = 0.25 V_{in}$

$\Delta\phi = \frac{\pi}{4} \rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$

$$\Delta t = \frac{\Delta\phi}{2\pi} T = \frac{\pi}{4} \frac{(200\,ms)}{2\pi}$$

$$= \frac{200}{8} = 25\,ms$$



2/ (a) $$G = \frac{\left(\frac{1}{Z_L} + \frac{1}{Z_R}\right)^{-1}}{Z_{C_1} + Z_{C2} + \left(\frac{1}{Z_L} + \frac{1}{Z_R}\right)^{-1}} = \frac{\left(\frac{1}{j\omega L} + \frac{1}{R}\right)^{-1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} + \left(\frac{1}{j\omega L} + \frac{1}{R}\right)^{-1}}$$

(b) $$Z_{in} = Z_{eq} = Z_{C_1} + Z_{C2} + \left(\frac{1}{Z_L} + \frac{1}{Z_R}\right)^{-1}$$

$$= \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + \left(\frac{1}{j\omega C} + \frac{1}{R}\right)^{-1}$$

(c) $Z_{out} = Z_{TH} = \frac{Z_1 Z_2}{Z_1 + Z_2}$ if it "looks like" a v-divider

$$= \frac{\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right)\left(\frac{1}{j\omega L} + \frac{1}{R}\right)^{-1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + \left(\frac{1}{j\omega L} + \frac{1}{R}\right)^{-1}}$$

3/ (a) $|G| = \sqrt{G^{+} G} = \left[\frac{R}{(R+R_L) - j(\omega L)} \frac{R}{(R+R_L) + j(\omega L)}\right]^{1/2}$

$$= \left[\frac{R^2}{(R+R_L)^2 + (\omega L)^2}\right]^{1/2}$$

(b) $G = \frac{R}{(R+R_L) + j\omega L} \cdot \frac{(R+R_L) - j\omega L}{R+R_L - j\omega L} = \frac{R(R+R_L) - j R\omega L}{(R+R_L)^2 + (\omega L)^2}$

$\tan\theta = \frac{\text{Im part}}{\text{Re part}} \rightarrow \theta = \arctan\left(\frac{-\omega R L}{R(R+R_L}\right)$

$$\theta = \arctan\left(\frac{-\omega L}{R+R_L}\right)$$

(c) $\lim_{\omega \to 0} |G(\omega)| = \left[\frac{R^2}{(R+R_L)^2 + 0}\right]^{1/2} = \frac{R}{R+R_L}$

$\lim_{\omega \to \infty} |G(\omega)| = \left[\frac{R^2}{(R+R_L)^2 + \infty}\right]^{1/2} = 0$

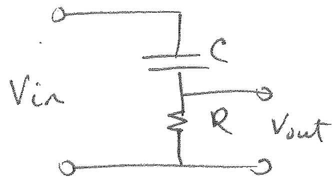
(d) this is a lp filter

$G \rightarrow 0$ in high frequencies

but it's not a good lp filter since $G < 1$ at $\omega \rightarrow 0$

4/(a) This is a high pass filter $G \to 1$ at high ω.
$G \to 0$ low ω

(b) A typical hp filter is an RC circuit. (There are other hp filters.)



Here, $|G| = \left[\dfrac{1}{1 + \left(\frac{\omega_{RC}}{\omega}\right)^2} \right]^{1/2}$ when $\omega = \omega_{RC}$, $|G| = \frac{1}{\sqrt{2}} = 0.707$

So 200 Hz is f_{RC}

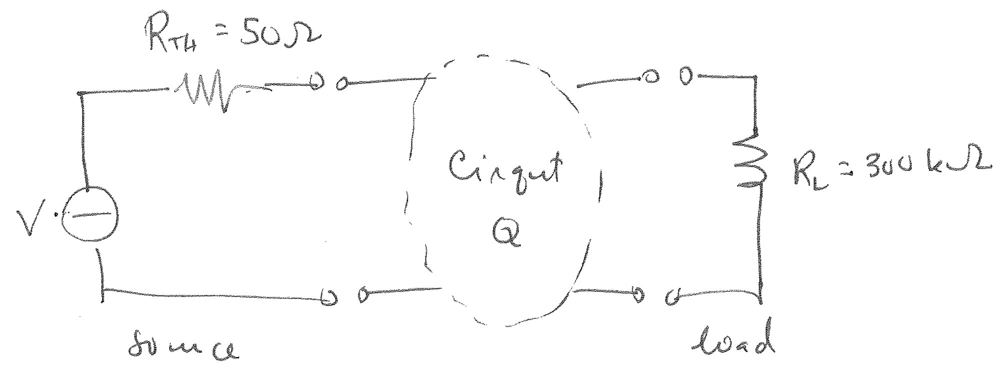
$$2\pi f_{RC} = \omega_{RC} = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi (200\,Hz)}$$

for $R = 1 k\Omega$ $C = 0.8 \mu F$

(Can choose any RC combination... but RC choice will determine Z_{in} + Z_{out} for this filter.)

5/



(a) Z_{in} of circuit Q $\gg$ Z_{TH} of source circuit.

$5\,k\Omega \gg 50\Omega$

so Z_{in} is $100 \times Z_{out}$.

Z_{out} of circuit Q $\ll$ Z_L of load

$3\,k\Omega \ll 300\,k\Omega$

[illegible] $100\times$ smaller than Z_L

(b) They're "good" because it will allow for

- using $G(\omega)$ expressions for each stage
- the V-signal transferred to each stage is maximized ($V \approx V_{TH}$ of each stage)
- therefore, largest signal transferred to output

If this isn't followed, will have signal attenuation.
i.e. it will be smaller.

6/ The impedance of a component defines the proportionality between the voltage + current of the component,

$$V = I Z \quad (1)$$

For a capacitor, its capacitance is $C = \frac{Q}{V}$. (2)

Rephrase Eqn (2) to be of the form of eqn (1).

We know that $\frac{dq}{dt} = i$

$$q = \int i\,dt$$

So (2) becomes

$$V(t) = \frac{Q(t)}{C} = \frac{\int i\,dt}{C}$$

Now let $i(t) = i_0 e^{j\omega t}$

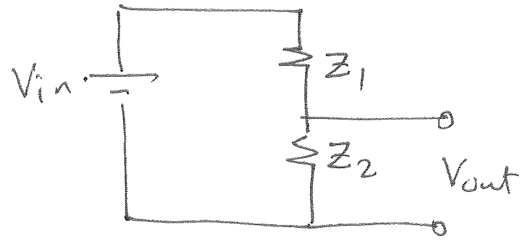
$$V(t) = \frac{1}{C} \int i_0 e^{j\omega t}\,dt$$

$$= \frac{1}{j\omega C} i_0 e^{j\omega t}$$

$$\boxed{V(t) = \frac{1}{j\omega C} i(t)}$$ Compare to Eqn (1)

therefore $\boxed{Z_C = \frac{1}{j\omega C}}$

7/



$V_{TH} = V_{out}$ of unloaded divider. Therefore, $V_{TH} = V_{out} = V_{Z_2}$

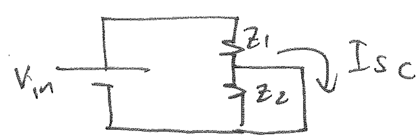
K. loop: $+V_{in} - IZ_1 - IZ_2 = 0$

$$I = \frac{V_{in}}{Z_1 + Z_2}$$

Ohm's law: $V_{Z_2} = IZ_2 = \dfrac{V_{in} Z_2}{Z_1 + Z_2}$

therefore $V_{TH} = \dfrac{Z_2}{Z_1 + Z_2} V_{in}$

now $Z_{TH} = \dfrac{V_{TH}}{I_{sc}}$ where



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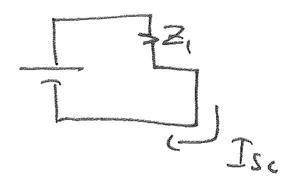

so $I_{sc} = \dfrac{V_{in}}{Z_1}$

$$Z_{TH} = \frac{Z_2 V_{in}}{Z_1 + Z_2} \frac{Z_1}{V_{in}}$$

$$Z_{TH} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

8/ • linear scales are useful if your data set spans only 1 or 2 orders of magnitude.

For example, (0.1, 0.2 .... 1 ... 10.1) } we should be able to see 0.1 + 10.1 both easily

or (1010, 1020 ... 2050 ... 5405) } or 1010 + 5405 easily

• Log scales are useful if your data set spans several orders of magnitude

For example, (0.1, 1, 10, 100, 1000, 10,000)
Plotting such a data set on a linear scale, we'd lose any sense on what's going on at the 0.1 + 1 region if our range = $0 \rightarrow 10^4$.

• In addition, log scales are useful for determining exponential or power-dependences.

For example, if $g = h^3$ or $\frac{V}{V_0} = e^{-t/RC}$

$$\underbrace{(\ln g)}_{\text{on y-axis}} = 3 \underbrace{(\ln h)}_{\text{on x-axis}}$$

will give a line w/ slope of 3

$$\underbrace{\ln\left(\frac{V}{V_0}\right)}_{\text{plot on y}} = \underbrace{\frac{-t}{RC}}_{\text{plot t on x}}$$

will give a line w/ slope of $-\frac{1}{RC}$

9/ • RLC filter has 3 possible outputs : V_C, V_R, V_L

Two of these (V_C, V_L) can have gain > 1 at the resonant frequency.

RC bandpass only has one possible output, with $g < 1$.

• RLC will have a phase shifted output at ω_{LC}

RC bandpass <u>isn't</u> phase shifted at ω_{RC}