

## 6 RC filters

In this lab, we

- measure the frequency response of a circuit
- explore the meaning of “characteristic frequency”
- identify the characteristic behavior of low-pass and high-pass filters
- compare theoretically expected and measured behaviors

### 6.1 Introduction

The RC circuits in this lab can be modeled as AC voltage dividers. Figure 6.1 shows a generalized voltage divider.

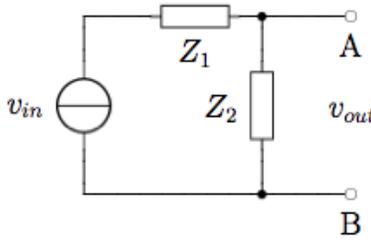


Figure 6.1: A generalized voltage divider constructed of two components with impedances  $Z_1$  and  $Z_2$ . The output is between the terminals A and B.

Components  $Z_1$  and  $Z_2$  can be any combination of elements. For a generic input voltage,  $v_{in}$ , the output voltage is given by our voltage-divider equation as

$$v_{out} = \frac{Z_2}{Z_1 + Z_2} v_{in}.$$

Since the impedances  $Z_1$  and  $Z_2$  may depend on the input frequency of our signal, it is helpful to write that explicitly. Thus, the gain of our circuit is

$$G(\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)} \quad (6.1)$$

We now write the input,  $v_{in}(t)$ , and output,  $v_{out}(t)$ , explicitly as sinusoidal signals,

$$\begin{aligned} v_{in}(t) &= V_{in} e^{j(\omega t + \phi_{in})} \\ v_{out}(t) &= V_{out} e^{j(\omega t + \phi_{out})} \end{aligned}$$

The quantities  $V_{in}$  and  $V_{out}$  are the amplitudes of the signals, while the angular frequency  $\omega$  is  $2\pi f$ . The two phases,  $\phi_{in}$  and  $\phi_{out}$ , determine the value of the signals at time  $t = 0$ . The gain of our circuit is then

$$G = G_0 e^{j\Delta\phi} \quad (6.2)$$

with an amplitude  $G_0 = V_{out}/V_{in}$  and phase  $\Delta\phi = \phi_{out} - \phi_{in}$ .

We now consider the specific case of RC filters. Consider the two filters shown in Fig 6.2. To analyze these, we identify  $Z_1$  with the resistor or the capacitor and  $Z_2$  with the opposite component. As discussed in our text, the characteristic frequency of these circuits is

$$\omega_{RC} = \frac{1}{RC} \quad (6.3)$$

Applying equation 6.1 to the low pass filter, we find that the gain is

$$G_{lp} = \frac{1}{1 + j\frac{\omega}{\omega_{RC}}} \quad (6.4a)$$

$$G_{lp}(\omega \ll \omega_{RC}) = 1 \quad (6.4b)$$

$$G_{lp}(\omega \gg \omega_{RC}) = -j\frac{\omega_{RC}}{\omega} \quad (6.4c)$$

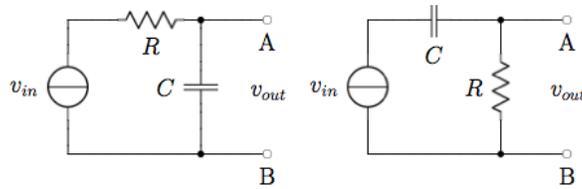


Figure 6.2: The left circuit is a low-pass filter. The right circuit is a high-pass filter.

where the last two equations describe the gain in the low and high frequency limits.

Applying equation 6.1 to the high pass filter, we find that the gain is

$$G_{hp} = \frac{1}{1 - j\frac{\omega RC}{\omega}} \quad (6.5a)$$

$$G_{hp}(\omega \ll \omega_{RC}) = j\frac{\omega}{\omega_{RC}} \quad (6.5b)$$

$$G_{hp}(\omega \gg \omega_{RC}) = 1 \quad (6.5c)$$

where the last two equations describe the gain in the low and high frequency limits.

## 6.2 Note taking reminders

By this time, it's expected that you have an idea of how to document your labwork. This is the last lab where there will be explicit instructions on how to take notes.

- *Update the table of contents.*
- *Write the header for this experiment.*
  - *Write the title.*
  - *Write the names of the person(s) you're working with. Write today's date.*
  - *Write the purpose.* Skim this lab. Write 1-3 sentences about what you'll be working on. It can be similar to what written in the manual – but certainly not word-for-word.

## 6.3 Low pass filter

We investigate the frequency response of the RC circuit when it is in the low pass configuration, as shown in Figure 6.3

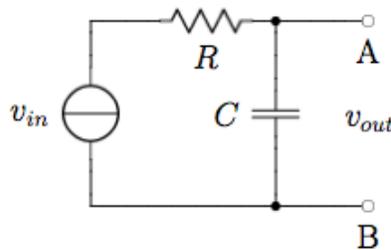


Figure 6.3: An RC circuit in the low pass configuration. Use a  $0.0047\mu\text{F}$  capacitor and a  $15\text{k}\Omega$  resistor.

This lab follows the standard process of many lab experiments. We (1) do a brief analysis that involves a calculation or two to guide our experiment, (2) build the circuit and setup the measurement, (3) collect data, and (4) analyze the data. It is common that while doing analysis it becomes apparent that you have to collect more data.

- *Start a new section.* Title it “Low pass filter.”

- *Describe the experiment.* Title this “Experimental methods and setup.”
  - Draw a circuit diagram of the low pass filter. Use the values given in the figure.
  - Calculate the characteristic frequency  $f_{RC}$  of the circuit. Keep in mind that  $\omega = 2\pi f$ .
  - Skim the rest of this section. Write 1-3 sentences about what you’ll do.
  - Make sure to include a list of equipment (names, what they’re meant to do, where they connect, ...).
- *Build the circuit.* Make the orientation of your components look like the diagram.
- *Setup your measurement.*
  - Use the function generator to supply the input signal  $v_{in}(t)$ . Use a sine wave with a peak-to-peak voltage of 10V. Pay attention to the load setting. Most of what we measure in this lab are considered high-impedance loads.
  - Put the input signal on Ch1. Use a BNC-T to split the signal from function generator to go the divider and to Ch1 simultaneously. The function generator is connected directly to the oscilloscope, so the probe setting should be 1x.
  - We’ll be using a probe to measure the output voltage. Attach the probe to Ch 2, then check the probe compensation. See notes below.
 

Once you’ve checked the probe compensation, connect the probe to your filter’s output terminals. Make sure the probe and input signal share the same ground (negative terminal). Check the probe setting for Ch 2. This is a 10x probe. That is, it attenuates the signal by 10x.
  - Check the other settings on the oscilloscope: Set the coupling to DC for both signals. Make sure the trigger source is Ch1 and set a reasonable trigger level.
- *Measure the frequency response of the low pass filter at  $f_{RC}$ .* Record this under “Data.”
  - With the cursor, measure the peak-to-peak voltages of  $v_{in}$  and  $v_{out}$ .
  - With the cursor, measure the time shift  $\Delta t = t_{out} - t_{in}$ , then calculate the phase shift  $\Delta\phi$ . Pay attention to whether this is a positive or negative phase shift. If you’re not sure, lookup the graph of  $\cos(x)$  and compare it to  $\cos(x - \pi/3)$ .
  - With the measure function, determine the peak to peak voltages of  $v_{in}$  and  $v_{out}$ . With the measure function, determine the phase shift of the output relative to the input. Compare these values with the ones you found using the cursor. If they’re consistent, use the measure function for the remainder of the lab. Keep in mind that measure will fail to work properly if the display shows a clipped signal, or is too small. It’s also a problem at low frequencies ( $< 1\text{Hz}$ ).
  - Decide on the uncertainty for your measurements. See notes below.
  - Print out the display. Label the input and the output signals.
- *Measure the frequency response for a range of frequencies.* Investigate frequencies 2 decades above and below the characteristic frequency. For example, if your characteristic frequency is 10Hz, go from 0.1Hz to 1kHz.
  - When working with frequencies that span a few decades, it’s best to use a log scale. We also want data points that uniformly fill on a log scale. Try the following frequencies 1Hz, 2Hz, 3Hz, 6Hz; then 10, 20, 30, 60; then 100, 200, and so on. See notes below for more details.
  - Measure  $v_{in}$ ,  $v_{out}$ , and  $\Delta\phi$ . Use the measure function. Note units and uncertainties (keep an eye on this, it might be different at 1Hz versus 10kHz). Record your data and uncertainties in a table. Create a calculated column for  $G_0$ .

measured quantities				computed quantities
$f(\text{Hz})$	$v_{in}(\text{V})$	$v_{out}(\text{V})$	$\Delta\phi$	$G_0 =  v_{out}/v_{in} $

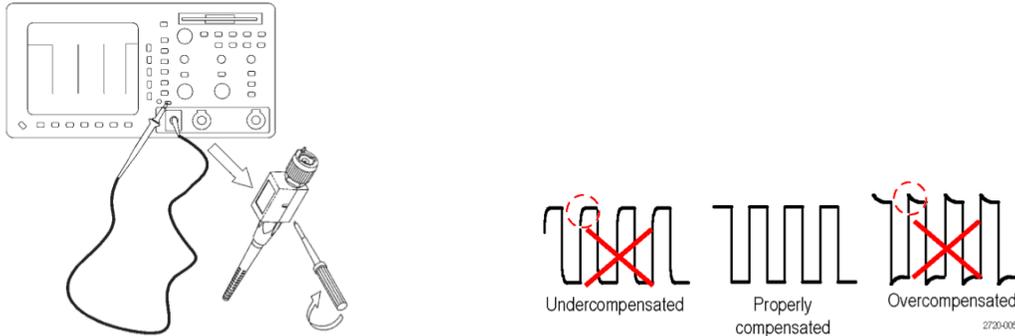
- Plot (preview) your data as you go. Look at  $G_0$  as a function of frequency  $f$  (on a log scale). Add points where it necessary. Once you have good dataset for  $G_0$ ,

- plot  $\Delta\theta$  as a function of  $f$ . Add points where it necessary.
- Print out datasets and both plots.
- *Analyze your data.*
  - Determine the theoretically expected behavior of  $G_0(\omega)$ . Start with Equation 6.4a. Determine an expression for  $G_0$ . Calculate the expected values of  $G_0$  for your frequencies.
  - Determine the theoretically expected behavior of  $\Delta\phi(\omega)$ . Start with Equation 6.4a. Determine an expression for  $\Delta\phi$ . Calculate the expected values of  $\Delta\phi$  for your frequencies.
  - Compare your measured values with the theoretically expected values. Plot your measured  $G_0$  with error bars. Plot the theoretically expected values on the same graph. Plot your measured  $\Delta\phi$  with error bars. Plot the theoretically expected values on the same graph.
  - Why is this called a low pass filter? Support your answer with your experimental observations.
  - What happens at  $f_{RC}$  for  $G_0(\omega)$  and  $\Delta\phi(\omega)$ ? Give specific behaviors and values.
  - To what extent does the theory describe your data? Support your answer with experimental observations.

### Notes on probe compensation

Attach the BNC end of the probe to Ch 2 input.

Attach the probe tip to the probe compensation test point on the oscilloscope front panel. Attach the grounding clip also.



Find the signal. Autoset is fine. It will be a 5 Vpp, 1kHz square wave.

Adjust the trimmer in the probe to obtain a waveform display that has flat tops with no overshoot or rounding.

### Notes on uncertainties

When reporting uncertainties, consider:

1. You. Did you eyeball the measurement or set the cursor yourself? How well do think you can do this? To within 1%, 2%, 5%, 10% or 20%?
2. Measurement fluctuation. What's the range of the fluctuation? If you actually take several measurements, report the mean and the standard deviation.
3. Instrument accuracy
4. Experimental design

Use the largest contribution to the uncertainty. Using the cursor versus using the measurement function will give different uncertainties.

There's no need to argue whether the uncertainty is 1%, 2% or 3%. At this point, it's expected that you can reasonably report whether the uncertainty is 2%, 5%, 10% or

20%. Keep in mind that the uncertainty may also change depending on the scale of your measurements. For example, it might be higher at low voltages than high voltages.

### Notes on the log scale

Analog circuits must operate properly over several decades (factors of 10) in frequency and amplitude. Circuits are expected to behave the same at, for example, 100kHz and at 10MHz, as well as at 0.10V and 10V. Therefore, it is essential to be able to characterize and display circuit behavior over decades.

When working over decades, linearly spaced data points and linear plots aren't effective. For example, if we want to look at the range of 1 Hz to 10MHz using linearly spaced data points, behavior from 1 to 100Hz will be unknown unless we take over 10,000 data points.

In this and subsequent labs, select where you take data so that the points uniformly fill several decades on a log scale. If you want  $n$  points per decade, choose  $10^{1/n}$  and integer powers thereof. For example, in frequency measurements with 4 points per decade, you could take data at  $10^{1/4}$ ,  $10^{2/4}$ ,  $10^{3/4}$ , and  $10^{4/4}$  . . . , which are at 1.78Hz, 3.16Hz, 5.62Hz, and 10Hz. Repeat for the next decade: 17.8Hz, 31.6Hz, 56.2Hz and 100Hz. When viewed over several decades, great precision is not necessary, so 1Hz, 2Hz, 3Hz, 6Hz, 10Hz, 20Hz, 30Hz, 60Hz and 100Hz . . . will work just fine.

## 6.4 High pass filter

Consider the same RC circuit. This time take the output is across the resistor. Switch the component positions to make sure the probe ground stays at ground.

- *Start a new section.* Title it “High pass filter.”
- *Describe the experiment.* Title this “Experimental methods and setup.”
  - Draw a circuit diagram. Include values.
  - Calculate the characteristic frequency of the circuit. Keep in mind that  $\omega = 2\pi f$ .
  - Skim the rest of this section. Write 1-3 sentences about what you'll do.
  - Make sure to include a list of equipment (names, what they're meant to do, where they connect, . . .).
  - A lot of this is similar to the low pass filter. You're welcome to just note the differences (and rewrite the whole thing).
- *Build the circuit.*
- *Setup your measurement.*
  - Use the function generator to supply the input signal  $v_{in}(t)$ . Monitor the input signal on Ch1. Use a probe to measure the output signal on Ch 2. Re-check the probe compensation. Once you've checked the probe compensation, connect the probe to your filter's output terminals. **Make sure the probe and input signal share the same ground.** Re-check the trigger and coupling settings on the oscilloscope.
  - Need more detail? See the low pass filter setup.
- *Measure the frequency response of this filter at the characteristic frequency.* Same as the low pass filter.
- *Measure the frequency response for a range of frequencies.* Same as the low pass filter.
- *Analyze your data.* Same as the low pass filter. You'll need to use Eqn 6.5a, and substitute in the words “high pass” for “low pass” where appropriate.

## 6.5 Design a low pass filter (optional)

One of the applications of a low pass filter is for audio equipment. It's common practice to send the low frequency (20-100 Hz) signals to a dedicated speaker (a woofer).

- Design a low pass RC filter that has a characteristic frequency of approx 100-200Hz Hz. Use a  $50\Omega$  resistor.

- Try it out. Use the computer's headset jack, a 3.5mm audio plug with its wires split, your RC filter, and a headset with its wires split. (You might remember this lab from Physics 4!)

The filters for audio systems are more complex than a series RC circuit. The idea of what the filter does, and how it does it, is the same. Other details such as how fast the signal is attenuated with increasing frequency, whether there's signal loss or gain at the input/output of the filter, or how to account for any phase shifts in the signal, certainly differs.

## 6.6 Conclusions or summary

- Write conclusions and/or a summary. Label this "Conclusions" or "Summary."
  - Write one paragraph on what you found. Be specific by providing values or in describing behavior.
  - Write one paragraph on things for next time: what you would do differently next time? or how to improve measurements or analysis? what other measurements should we take? what should you really remember to do?