# Frequency Response of RC High Pass and RL Low Pass Filters

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This experiment examined the relationship between gain and phase shift with the input frequency of an RC high pass filter and an RL low pass filter. This was done by connecting an AC function generator with a varying sinusoidal voltage to an oscilloscope along with the output voltage from the circuits. We concluded that the RC high pass filter experiences a maximum gain of $20\text{dB}\log|G| = 0.65585 \pm 0.3477$ for the highest input frequencies and the RL low pass circuit had a maximum gain of $20\text{dB}\log|G| = 0 \pm 0.05489$ for the lowest frequencies.

## 1. INTRODUCTION

High pass and low pass filters have important audio uses. For example, high pass filters can be used in microphones to remove low frequency signals while preserving the quality of a speaker's voice. Low pass filters can be used when it is desired to silence sound. Cars containing piston engines are often equipped with a low pass filter to cut out high frequencies and act as a silencing mechanism [2].

We expect to find that the high pass filter reduces the voltage of lower inputs while experiencing the greatest gain for high frequency signals, and that the low pass filter will experiences the greatest gain for low frequencies while attenuating the higher input signals.

In this lab we explored the characteristic behaviors of an RC high pass circuit and an RL low pass circuit. We measured the output voltages and phase shifts of these circuits for input signals with varying frequencies. What we were specifically looking to find was the gain for each of the circuits as a function of frequency. We also determined the phase shift at different frequency values for both circuits.

To achieve these goals, we built an RC high pass circuit and an RL low pass circuit and quantitatively measuring their output signals. By connecting both the output signal from the circuit along with the direct signal to an oscilloscope, we can compare the two for differences in voltage and the phase differences. Our expectations for the way that the high pass and low pass filters would treat signals of varying frequency were confirmed: the high pass filter attenuates low frequencies and allows high frequency signals, and low pass filters attenuate high frequency signals and allow low frequencies to pass through.

## 2. THEORY

High pass filters are known to allow input voltages from higher frequencies through while preventing lower frequencies from getting through. Low pass filters, as one would expect, act in the opposite manner. This leads us to expect the voltage gain to be greater for higher frequencies when examining the high pass filter, and for lower frequencies when looking at the low pass filter.

For this experiment, we will be analyzing the gain using the equation $20\text{dB}\log|G|$ where the gain $G$ is measured as:

$$G = \frac{V_{out}}{V_{in}}. \tag{1}$$

The theoretical gain $|G|$ and phase shift $\Delta\phi$ for the RC high pass circuit are:

$$|G| = \left[1 + \left(\frac{1}{R\omega C}\right)^2\right]^{-1/2} \tag{2}$$

$$\Delta\phi = \arctan\left(\frac{-1}{R\omega C}\right) \tag{3}$$

where $R$ is the resistance, $C$ is the capacitance, and $\omega$ is frequency of the input signal.

For the RL low pass circuit, the theoretical equations for gain $|G|$ and phase shift $\Delta\phi$ are:

$$|G| = \left(\frac{R}{R+R_L}\right)\left[1 + \left(\frac{\omega}{\omega_{RL}}\right)^2\right]^{-1/2} \tag{4}$$

$$\Delta\phi = \arctan\left(\frac{-\omega}{\omega_{RL}}\right) \tag{5}$$

where $R$ is the resistance of the resistor, $R_L$ is the resistance of the inductor, $\omega$ is the frequency of the input signal, and $\omega_{RL}$ is the characteristic frequency calculated by $\omega_{RL} = \frac{R+R_L}{L}$.

## 3. EXPERIMENT

The system for measuring the phase shift and output gain characteristics of the RC high pass and RL low pass circuits consisted of an AC function generator, an oscilloscope, a resistor, and either a capacitor or inductor depending on the circuit. These circuits are shown in Figures 1 and 2.

Through the use of the oscilloscope, we compared the input voltage from the AC function generator with the measured voltage across the resistor in the circuit. We measured the peak to peak voltage from both sources as well as the time delay between the two signals by using the oscilloscope.

We estimated our uncertainty for each measurement based on the clarity of the oscilloscope reading. Each

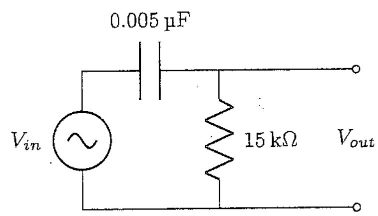


FIG. 1: RC High Pass Circuit. $V_{in}$ comes from the function generator and $V_{out}$ leads to the oscilloscope. The Capacitor has capacitance $C = 0.005\mu F \pm 5\%$ and the Resistor has resistance $R = 15\text{k}\Omega \pm 1\%$.

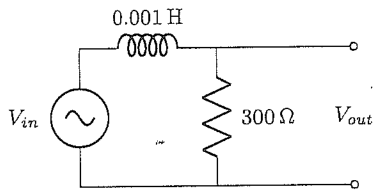


FIG. 2: RL Low Pass Circuit. $V_{in}$ is the source from the AC function generator and $V_{out}$ leads to the oscilloscope. The Inductor has inductance $L = 0.001\text{H} \pm 5\%$ and the Resistor has resistance $R = 300\Omega \pm 1\%$.

measurement was estimated to have either 2%, 5%, or 10% error depending on the precision of the output signal. 10% error was used if the signal was difficult to read, 2% was used if the signal was clear, and 5% error was used if the signal was somewhat clear but not quite difficult to read.

## 4. RESULTS

Figure 3 provides an example scope trace for the RC filter. The raw data for this experiment was gathered from similar oscilloscope readings. A sampling of the collected data is given in Table I.

| RC High Pass | | | RL Low Pass | | |
|---|---|---|---|---|---|
| $f$ (Hz) | $G$ | $\Delta\phi$ | $f$ (Hz) | $G$ | $\Delta\phi$ |
| 5 | 0.0039 | -1.571 | 4.81 | 1.00 | 0.003 |
| 2122 | 0.6923 | -0.794 | 48,000 | 0.9823 | -0.064 |
| 985,000 | 1.0784 | 0.0914 | $4.8\times10^6$ | 0.0887 | -1.511 |

TABLE I: Selected data points for the measured results of the RC high pass filter and RL low pass filter.

The phase shift was calculated from the raw data in order to convert the quantity from time delay to a phase shift using the following equation: $\Delta\phi = 2\pi f \Delta t$. The uncertainty in $\Delta\phi$ is $2\pi f \delta\Delta t$ where $\delta\Delta t$ is the uncertainty in the time difference as observed from acquiring the raw data. The phase plots for both circuits are shown below in Figures 4 and 5.

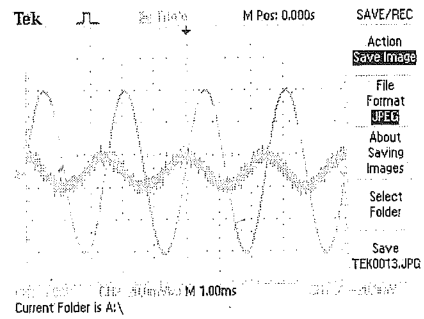


FIG. 3: A sample oscilloscope reading showing the scope trace for the RC high pass filter with a low frequency input. The functions show the input signal in yellow and output in blue for a low frequency signal.

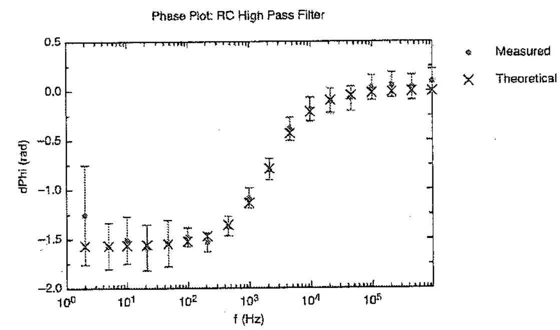


FIG. 4: RC high pass phase plot with theoretical expected values. The measured values also include error bars.

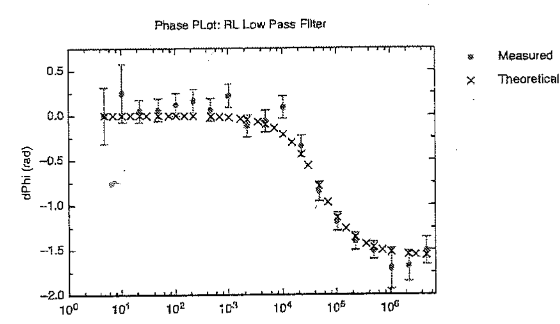


FIG. 5: RL low pass phase plot including theoretical expected values.

From our measurements of $V_{\text{in}}$ and $V_{\text{out}}$, we calculated the amplitude of the gain $|G| = \frac{V_{\text{out}}}{V_{\text{in}}}$. The uncertainty in $|G|$ is $\frac{\Delta G}{G} = \frac{\Delta V_{\text{out}}}{V_{\text{out}}} + \frac{\Delta V_{\text{in}}}{V_{\text{in}}}$. The Bode Plots in Figures 6 and 7 show the gain as the function $20\text{dB}\log|G|$ with uncertainty $\frac{20\delta G}{G\ln 10}$.

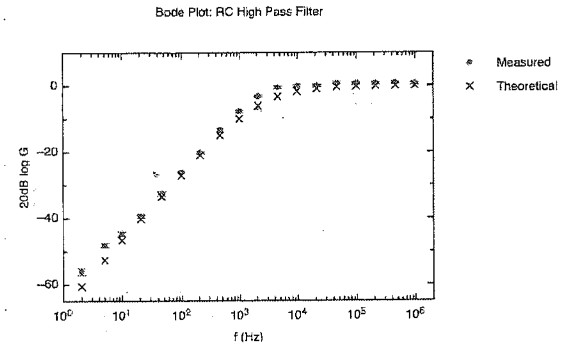


FIG. 6: RC high pass Bode plot with theoretical expected values.

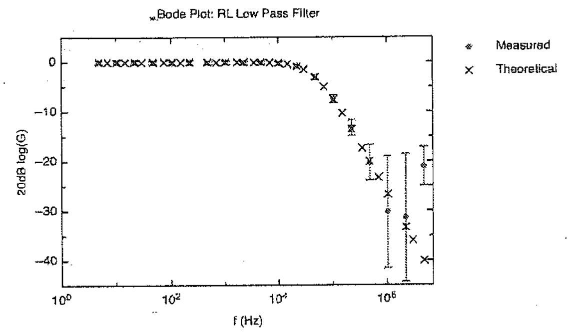


FIG. 7: RL low pass Bode plot with theoretical expected values.

## 5. DISCUSSION

Our results typically follow the theoretical expected values, although they are not entirely in agreement. Figure 4 shows that our measurements completely agree with the theoretical calculations since each of the values is within the margin of error of the expected result.

In Figure 5 we can see that the observed values are slightly off of the expected values for a small number of input frequencies. However, both sets of points follow the same trend and are only off slightly in certain areas. One possible explanation for this discrepancy is that we may have underestimated the margin of error. The same explanation can be made for Figure 6.

For the data shown in Figure 7, our measurements agree with the theoretical values at every frequency except for the final point. We came up with the same results shown upon remeasuring the data. This leads us to believe that there is something happening in the circuit or measuring devices that we have not fully accounted for.

The phase plots for both the RC and RL filters have inflection points at their respective characteristic frequencies of $f_{RC} = 2122$ Hz and $f_{RL} = 48,000$ Hz. Both circuits also experience a total phase shift of $\frac{\pi}{2}$ from low to high frequencies, but in opposite directions. Figure 4 shows that the RC circuit expereinced a change in phase from $\frac{-\pi}{2}$ to 0 as frequency increased. Figure 5 shows that the RL circuit started with both signals in phase before dropping to a $\frac{-\pi}{2}$ phase shift for high frequencies.

The Bode plot for the RL filter showed no gain for lower frequencies and had a logarithmic decrease in gain for frequencies greater than the characteristic frequency of $f_{RL} = 48,000$ Hz. The Bode plot for the RC filter showed that lower frequencies have negative values for $20\text{dB}\log|G|$ that steadily increase to zero gain for frequencies above the characteristic frequency.

Our results show that high pass filters have no gain for higher frequencies, meaning that the input voltage is the same as the output voltage. The lowest frequencies have the smallest gain, increasing exponentially until it reaches zero gain. The phase starts at $\frac{-\pi}{2}$ for low frequencies and approaches zero as frequency increases with an inflection point at the characteristic frequency.

For low pass filters, lower frequencies have no loss in voltage while frequencies above the characteristic frequency have exponentially decreasing negative values for $20\text{dB}\log|G|$. The phase goes from 0 to $\frac{-\pi}{2}$ as frequencies increase with an inflection point at the characteristic frequency.

## 6. CONCLUSIONS

It was found that the RC high pass filter has the greatest gain for high frequency signals of $20\text{dB}\log|G| = 0.7 \pm 0.3$ Hz and the lowest gain for low signals at $20\text{dB}\log|G| = -56 \pm 1$. The phase shift for low frequencies started at $\Delta\phi = -1.6 \pm 0.2$ and progressed to $\Delta\phi = 0.03 \pm 0.13$ for the highest tested frequencies. The phase shift went from approximately $\frac{-\pi}{2}$ to 0 as frequency increased.

The RL low pass filter had the greatest gain of $20\text{dB}\log|G| = 0.00 \pm 0.05$ for the lowest frequency and had the lowest at $20\text{dB}\log|G| = -30 \pm 10$ for the highest tested frequency. The phase shift for lower frequencies started at $\Delta\phi = 0.0 \pm 0.3$ and decreased to $\Delta\phi = -1.7 \pm 0.2$ for higher frequencies. This change was from approximately 0 to $\frac{-\pi}{2}$ as frequency increased.

[1] Meyer, C. A. Basic Electronics: An Introduction to Electronics for Science Students. Raleigh, N.C., 2010. Print.

[2] Watkinson, John. The Art of Sound Reproduction. Woburn, MA: Focal, 1998.