7 RLC and bandpass circuits

In this lab, we

- measure the frequency response of an RLC circuit.
- investigate the phenomenon of resonance.
- build and measure the frequency response of a band-pass filter.
- become more familiar with input and output impedances.

7.1 Introduction

RLC filters

RLC circuits play a large role in the modern world. An important use is in receivers, where they select out a particular frequency. There are numerous configurations of RLC circuits. In this lab, we consider a series RLC circuit as shown in Figure 7.1
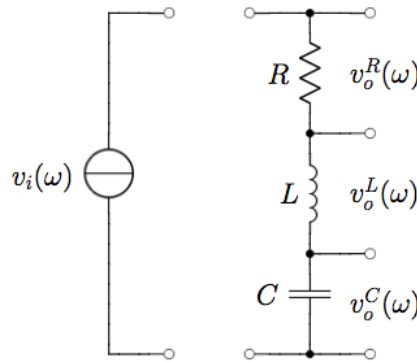

Figure 7.1: A series RLC circuit

The series RLC circuit has three possible output voltages. These are the three voltages across the three components of the circuit. We can use an impedance analysis to determine the gain of the circuit. The voltage divider analysis, given by Equation 6.1 of the previous lab, is also applicable to RLC circuits. Again, the gain depends on which component is used for the output.

$$G_X(\omega) \quad = \quad \frac{Z_X(\omega)}{Z_{tot}(\omega)} \tag{7.1}$$

The total impedance for this circuit is

$$
\begin{aligned}
Z_{tot} \quad &= \quad Z_R + Z_C + Z_L \\
&= \quad R + j\omega L + \frac{1}{j\omega C} = R\left[1 - j\frac{\omega_{RC}}{\omega}\left(1 - \frac{\omega^2}{\omega_{LC}^2}\right)\right]
\end{aligned}
$$

where we have defined the following characteristic frequencies,

$$\omega_{RC} \quad = \quad 1/RC \tag{7.2a}$$
$$\omega_{LC} \quad = \quad 1/\sqrt{LC} \quad \text{is the resonant frequency.} \tag{7.2b}$$

If we look at the gain to be across the resistor,

$$G_R(\omega) \quad = \quad \frac{R}{Z_{tot}} = \frac{1}{1 - j(\omega_{RC}/\omega)(1 - \omega^2/\omega_{LC}^2)}.$$

On the other hand, if we look at the gain to be across the capacitor,

$$G_c(\omega) \quad = \quad \frac{Z_c}{Z_{tot}} = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_{LC}}\right)^2\right] + j\frac{\omega}{\omega_{RC}}}. \tag{7.3}$$

A similar expression can be found for the gain across the inductor.

The resulting behavior of an RLC circuit is more complex than RC and RL circuits, as the circuit exhibits resonant behavior at a characteristic frequency. The resonance frequency depends on the inductance and capacitance of the circuit and is given by ω_{LC}.

Bandpass filters

There are many occasions when we need to couple one functional block of circuitry to another. In fact, it's hard to think of a situation where this is not necessary! In the case of a high-pass filter connected to a low-pass filter, we create a band-pass filter. Such a circuit will attenuate signal both above and below some characteristic frequency.

The new feature is that we need to worry about the input and output impedances of the two filters. In the case that the output impedance of the first stage is much smaller than the input impedance of the second filter,

$$|(Z_{\text{out}})_{\text{filter 1}}| \ll |(Z_{\text{in}})_{\text{filter 2}}|. \tag{7.4}$$

then the output is the product of the two individual gains

$$v_{\text{out}}(\omega) = G_{lp}(\omega)G_{hp}(\omega)v_{\text{in}}(\omega) \tag{7.5a}$$

$$= \left(\left[\frac{1}{1+(\omega/\omega_{RC})^2}\right]^{1/2} e^{j\phi_{lp}} \left[\frac{1}{1+(\omega_{RC}/\omega)^2}\right]^{1/2} e^{j\phi_{hp}}\right) v_{\text{in}}(\omega). \tag{7.5b}$$

where we substitute in the expressions for G_{lp} and G_{hp} in their exponential (or polar) forms. The above equation indicates that output amplitude is determined by a simple multiplication of the two gains, $|v_{\text{out}}| = |G_{lp}||G_{hp}||v_{\text{in}}|$; and that the phase of the output is the sum of the two phase shifts from each filter, $\phi_{\text{out}} = \phi_{lp} + \phi_{hp}$.

7.2 Reminders!

1. Circuit building: Are you using the buses? Does your layout look like the diagram? Are you minimizing the number of jumper wires?
2. Function generator: Is it set to "High-Z" or 50Ω?
3. Oscilloscope probe: Is the setting correct (1x, 10x ...)? Is the probe properly compensated?
4. Oscilloscope channels: Should you be using AC or DC coupling? (when in doubt, use DC)
5. Circuit board and oscilloscope probes: Are all of your grounds connected to the same point?
6. Note taking reminders. You're expected to have a Title, Header (Partner, Date, Purpose), Experimental Methods, Data, Analysis and Conclusion sections. Arrange them in some coherent way in your notebook.
7. For your data, keep in mind...

    (a) Include units and uncertainties on all measured values. Note how you determined these uncertainties.
    (b) Organize your data into tables. Graph your data as you go.
    (c) Graphs should have titles, error bars, axes titled and labeled with units, and be reasonably sized so that all the data, including error bars, can be easily seen.

7.3 RLC resonant circuit

We will investigate an RLC circuit configured to measure the voltage across the capacitor. Figure 7.2 provides a circuit diagram.

- Measure your inductor's resistance, R_L.
- Calculate the theoretical resonance frequency of your circuit. Give this both in rad/sec and Hz.
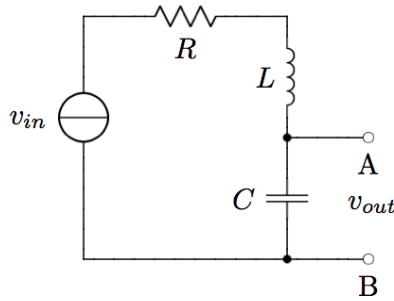
Figure 7.2: A series RLC circuit with an output across the capacitor. For this experiment, use a capacitor on the order of 0.01μ F and an inductor of 500mH. We will use the inductor's intrinsic resistance R_L and the resistor. Don't add any additional resistor to your circuit.

- Construct the RLC circuit. Take the output over the capacitor. Use a sine wave of a reasonable amplitude (say 5 V) as the input voltage.
- Measure the frequency response of your RLC circuit.

    Measure v_{out} and v_{in} over a frequency range that extends at least two decades above and below your calculated resonance frequency. Choose frequencies so that your points will be roughly equally-spaced on a logarithmic frequency axis. Choose more points near ω_{LC} to accurately map the behavior.

    For your first data point, try a frequency of about $\frac{1}{2}\omega_{LC}$. Use the measure function to measure amplitude. Use the cursors to determine phase the first time. Compare this value to the automated phase measurement. If they're within about 5% of one another, you're welcome to use the automated measurement.

    As you take data, pay attention to the phase measurement. Make sure it's reporting a value that makes sense. Keep in mind which direction is considered a positive phase shift. Also keep in mind that $-\pi/2$ is the same as $3\pi/2$.

Analysis

- Determine expressions for the amplitude and phase of the gain, $|G(\omega)|$ and $\Delta\phi(\omega)$. For amplitude, start from the general expression for gain (7.1), derive Equation 7.3, then determine its magnitude. For phase, start from Equation 7.3, rewrite it into the form $z = a + jb$, then determine $\phi(\omega)$.
- Create gain (amplitude) and phase response plots of your data. Error bars and error propagation are expected here.
- Include the theoretical functions to your plots. Take R_L into account.
- What are the magnitude and phase of the gain at the resonant frequency? at frequencies much larger and much smaller than the resonant frequency? Compare expected and measured values.
- Does your experimental data agree with what is expected theoretically? Write one paragraph of text.

7.4 Bandpass filter

In this section, we build a bandpass filter by taking the output of a high pass RC filter and putting it into a low pass RC filter with the same characteristic frequency. The bandpass filter we will investigate is shown in Figure 7.3.

- Calculate the characteristic frequencies of the two RC filters. Include uncertainties.
- Build your bandpass filter.
- Measure the frequency response of your bandpass filter. Keep in mind that this means both amplitude and phase. Explore a range of frequencies about 3 decades above and
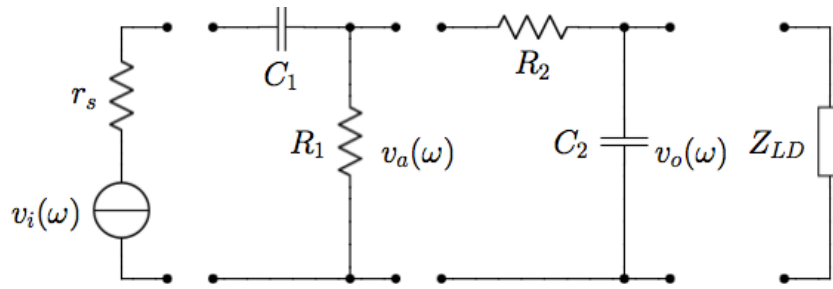
Figure 7.3: A bandpass filter. The Thevenin equivalent circuit for the source – in our case, a function generator – is on the left. This is followed by a high pass filter, with output v_a. The output v_a is connected to the low pass filter, with an output of v_o. Finally, The output from the low pass filter is connected to a load Z_{LD}. For the first stage (high pass filter), use $R_1 = 1.5\text{k}\Omega$ and $C_1 = 0.020\mu\text{F}$. For the second stage (low pass filter), use $R_2 = 30\text{k}\Omega$ and $C_2 = 0.001\mu\text{R}$.

below the characteristic frequency. Use a a sine wave of reasonable amplitude as the input voltage.

Analysis

- Describe the frequency response of your bandpass filter. Describe its behavior at the characteristic frequency, and in the two limits of $\omega \gg \omega_{RC}$ and $\omega \ll \omega_{RC}$ Why is it called a bandpass filter?
- Compare your measured frequency response with the theoretically expected response.
  - Create a plot of the amplitude of your gain. Include error bars. Plot the expected theoretical response atop your data.
  - Create a phase response plot of your data, include error bars. Plot the expected theoretical response atop your results.

- To what extent does the theory describe your data? Support your answer with experimental observations.

7.5 Bandpass filter revisited: input and output impedances

When coupling circuits together, we have to be mindful of the input and output impedances of each stage. To deliver the largest output signal to the following stage, the output impedance of the first stage (the driving circuit) must be much less than the input impedance of the second stage (the driven circuit), $Z_{\text{out}} \ll Z_{\text{in}}$. This is equivalent to saying that the the load impedance must be much larger than the Thévenin impendance, $Z_{\text{Th}} \ll Z_L$. The prior bandpass filter had values of R and C chosen to satisfy these conditions.

Here, we consider what happens when this condition isn't met.

- Build a bandpass filter where $R_1 = R_2 = 1.5\text{k}\Omega$ and $C_1 = C_2 = 0.020\mu\text{F}$.
- Calculate the characteristic frequencies of the two RC filters. Include uncertainties.
- Measure the frequency response of your bandpass filter. Keep in mind that this means both amplitude and phase. Explore a range of frequencies about 3 decades above and below the characteristic frequency. Use a a sine wave of reasonable amplitude as the input voltage.

Analysis

- Does the first bandpass filter satisfy the condition that $|Z_{\text{out}}$ of the first stage$| \ll |Z_{\text{in}}$ of the second stage$|$? Impedances here depend on the frequency, so calculate them in the case that $\omega = \omega_{RC}$.

- Does the second bandpass filter satisfy the condition that $|Z_{\text{out}}$ of the first stage$| \ll |Z$ of the second stage$|$? Impedances here depend on the frequency, so calculate them in the case that $\omega = \omega_{RC}$.
- Compare the frequency response of your two bandpass filters. How are they the same and how do they differ? It's best to plot them together.