

# Optimization of Baseball Swing Parameters For Three Levels of Play

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## Abstract

Hitting homeruns in baseball is a key factor to the sport: it provides a guaranteed run for the team, excitement for the fans, and bragging rights for the individual. This investigation adapts Hubbard<sup>i</sup> *et al*'s work to examine the optimal swing parameters resulting in maximum range at three different levels of play: Little League, high school, and college. The two parameters optimized in this study describe the bat's path to the ball:  $E$  the undercut distance between the center of the bat and ball; and  $Psi$ , the undercut angle. The purpose of this optimization is to determine what  $E$  and  $Psi$  values result in the maximum range of the batted ball.

A four-phase program was used to determine the optimal values for  $E$  and  $Psi$ . Phase I was added to Hubbard *et al*'s program because the experimental pitch data used in their study was not available for lower levels of play. Thus Phase I calculated the pitch angle using the height difference between the release point of the ball and the center of strike zone. Phase II consisted of the impact analysis of the pitched ball and bat. A flight simulation accounting for drag and lift forces constituted Phase III of the program. Phase IV carried out the optimization of the values  $E$  and  $Psi$  by running Phases II and III repeatedly until the largest range was found. This method was carried out six different times using two pitch speeds for each playing level's set of parameters.

It was found that at lower levels of play, meaning slower pitch speed and slower bat speed,  $E$  and  $Psi$  were larger than at higher levels of play. Thus Little Leaguers should swing up on the ball more than high school and college level players.

## Introduction

Athletes and coaches alike are constantly striving to improve performance whether it is at a recreational or professional level. Improvement has been sought after through increased training, trial and error of new mechanical processes, and even illegal substance abuse. However, a new method is beginning to develop called sports biomechanics, a field in which human movements are studied by applying physical principles and laws. This project uses biomechanics to investigate a particular baseball skill: hitting a pitched baseball as far as possible.

There have been several dozen papers published that analyze various aspects of baseball. Typically, the problem of the batted ball has been broken up into two parts: an impact analysis and a flight analysis. Historically, the flight of a baseball was investigated first by Briggs<sup>ii</sup> who answered the question, “Does a curveball really curve?” Briggs dropped balls spinning about a vertical axis through a horizontal wind tunnel and measured their deflection. He found that curveballs do in fact curve. Furthermore, the amount of deflection is proportional to the ball’s spin and translational velocity squared. Next, Achenbach<sup>iii</sup> discovered that the drag force acting on a ball is dependent on the Reynolds number ( $Re$ ) of the fluid and the roughness of the sphere. Reynolds number is the ratio of inertial forces to viscous forces and is directly proportional to the velocity of the fluid, air in this case. The  $Re$ <sup>iv</sup> can be used to determine whether the airflow is laminar, meaning smooth, or turbulent, meaning chaotically changing. Achenbach noticed that there was a critical  $Re$  where the drag constant dramatically decreased in magnitude. This property is known as the drag crisis and likely occurs at speeds typical of pitched or batted baseballs. In order to accurately simulate a baseball’s flight, drag and lift forces must be accounted for. Neither of these forces are constant and should be calculated dynamically.

Several groups have also studied the impact between the bat and baseball. Rex<sup>v</sup>, Watts and Baroni<sup>vi</sup>, and Always *et al*<sup>vii</sup> have all created models that examine the initial bat-ball collision and flight of the ball to predict its trajectory. Hubbard *et al* improved the process by using a more complex impact analysis based on a planar rigid-body impact methodology and a dynamic flight analysis. Their method also allows the direct calculation of the optimal parameters for maximum range given the initial conditions of the problem. Thus given the velocity vector of the bat, spin of the bat, velocity vector of the pitched ball, and spin of the pitched ball the optimal undercut distance,  $E$ , and undercut angle,  $Psi$ , can be found.  $E$  and  $Psi$  can be seen in Figure 1.

Hubbard *et al* found: “There is an optimal strategy for achieving maximum range. For a typical fastball the batter should undercut the ball by 2.65 cm and swing upward at an angle of 0.1594 radians.”<sup>i</sup> Furthermore, an optimally hit curveball will travel farther than an optimally hit fastball. This demonstrates the importance of spin on the range of a ball. A fastball has a negative angular velocity before it is hit and a positive angular velocity after it is hit. However, a curveball starts with positive angular velocity and its positive angular velocity is increased after it is hit. Positive angular velocity is important because it generates lift upward which holds the ball in the air longer allowing the ball to travel a farther horizontal distance.

This investigation is an adaptation of Hubbard *et al*’s work which examines how the optimal batting strategy changes for three different levels of play: Little League, high school, and college. For example, Little Leaguers use lighter bats, swing slower, and face slower pitched ball speeds than Major League players do. Thus it is logical that Little Leaguers should swing differently than

Major Leaguers do. This project identifies which variables are dependent on play level, quantifies these variables, and finally finds the optimum batting strategy for different levels of play.

## Methods and Calculations

### Identification of Parameters

As this project modifies Hubbard *et al's* existing MATLAB code used for professional baseball, the first step completed was to identify the parameters and variables that change at amateur levels of play. The significant difference between professional and amateur baseball players is the bat. Amateur players use aluminum bats which have a different coefficient of friction, axial radius of gyration, and transverse radius of gyration. Furthermore, the size of the bat and its linear velocity (swing speed) vary significantly depending upon the level of amateur play. The different bat properties used in this investigation are shown in Table 1.

**Table 1: Bat Properties**

Play Level	Professional	College	High School	Little League
Length (m)	0.813	0.883	0.785	0.732
Barrel Radius (m)	0.0350	0.0332	0.0284	0.0280
Mass (kg)	0.900	0.884	0.641	0.503
Axial Radius of Gyration (m)	0.0248	0.0332	0.0284	0.0280
Transverse Radius of Gyration (m)	0.2170	0.2732	0.2404	0.2443
Coefficient of Friction	0.50	0.35	0.35	0.35

Conveniently, the balls used in amateur and professional baseball are the same. Therefore the variables that change depending on play level are the ball's kinematic properties: velocity, angular velocity, and position which are shown in Table 2. This means that the lift and drag coefficients used to calculate the corresponding forces are the same as in professional baseball. Thus the portions of the program that calculate the lift and drag forces did not need to be modified. Finally, while there are many different types of pitches, this study only examines fastballs because Little Leaguers may not have developed the skills to throw other pitches.

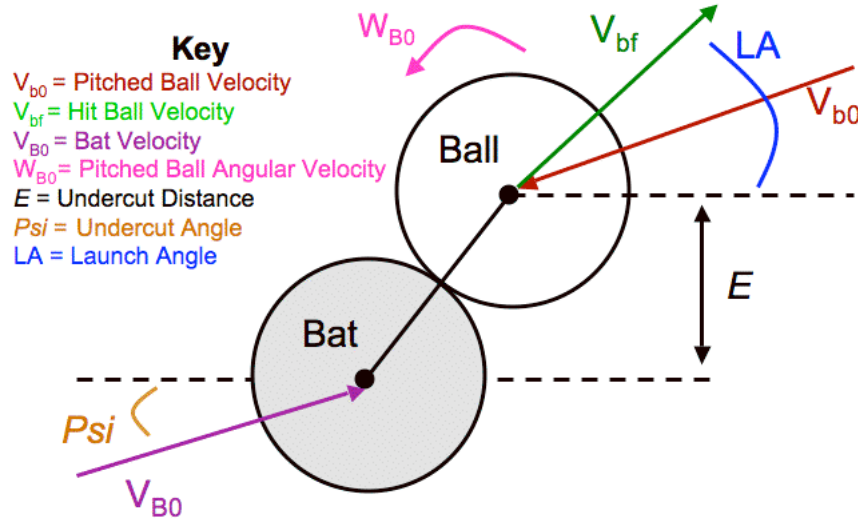
**Table 2: Ball Properties**

Play Level	Professional	College	High School	Little League			
Pitched Ball Velocity (mph)	94	90	80	70	60	50	40
Pitched Ball Angular Velocity (rad/s)	-200	-191	-170	-149	-128	-106	-85
Pitched Ball Angle (rad)	0.1556	0.1251	0.1353	0.1478	0.1660	0.1930	0.2591

In summary, both the bat and ball have two types of properties that describe them. There are parameters: mass, radius of gyration, etc. and variables: velocity, angular velocity, etc. The purpose of this investigation is to use the correct parameters for each level of play and then see how the changing variables affect the two parameters being optimized:  $E$  and  $Psi$ . Figure 1 defines the kinematic properties of the bat and ball. Note that topspin is positive in this coordinate system. Thus a fastball, which has backspin, has a negative angular velocity as seen in Table 2. Furthermore, the properties given in these tables are "typical"; meaning an average player of these levels would

use/experience these values. Thus, the next portion of the project involved determining the values given in the above tables through experimental measurement and research.

**Figure 1: Bat-Ball Kinematic Properties**



### Measurements

The impact analysis between the bat and ball uses the principles of momentum conservation and therefore requires the mass of the bat as a parameter. However, these calculations require the *effective* mass of the bat rather than its actual mass. This study assumes that the ball makes contact at the “sweet” spot, or center of percussion, of the bat. Thus, the mass of the bat striking the ball is not what it appears on a scale. Furthermore, the bat is being swung about its end. Thus the mass of the bat used in the calculations must take into account the mass, length, radius, and radius of gyration of the bat. These properties were measured for three different bats.

The first three properties were easily measured using a scale and meter stick. However, the radius of gyration was determined indirectly through experimentation. The radius of gyration is defined as:

$$k = \sqrt{\frac{I}{M}} \tag{1}$$

where  $I$  is the moment of inertia and  $m$  the mass of the object. Thus the radius of gyration of an object is really just a convenient way to write the moment of inertia of an object. The moment of inertia of an object about a given axis describes how difficult it is to induce an angular acceleration of the object about that axis. The transvers moment of inertia of an object can be determined by measuring the object’s period of oscillation. By beginning with the definition of torque:

$$\tau = I\alpha \tag{2}$$

the following equation for the moment of inertia about the knob of the bat can be derived:

$$I_{knob} = \frac{mgr_{cm}T^2}{4\pi^2}$$

(3)

where  $m$  is the actual mass of the bat,  $g$  the acceleration of gravity,  $r$  the radius of the bat, and  $T$  is the period. Using the parallel axis theorem<sup>viii</sup>, the moment of inertia about the sweet spot of the bat is:

$$I_{cm} = mr_{cm}^2 \left( \frac{gT^2}{4\pi^2 r_{cm}} - 1 \right) \quad (4)$$

To determine the period of each bat, the bat's knob was held in a string cradle so that the bat hung vertically and rotated about its end. The bat was then displaced slightly from its equilibrium position so that it swung back and forth. A photogate was used to accurately determine when the bat had completed one oscillation. Three trials were carried out for the three different bats. In each trial, the bat was allowed to complete 14 periods. These periods were averaged to limit uncertainty. Figure 2 is a graph of the average moment of inertia versus mass for each of the three bats.

**Figure 2: Comparison of Moment of Inertia of 3 Bats**

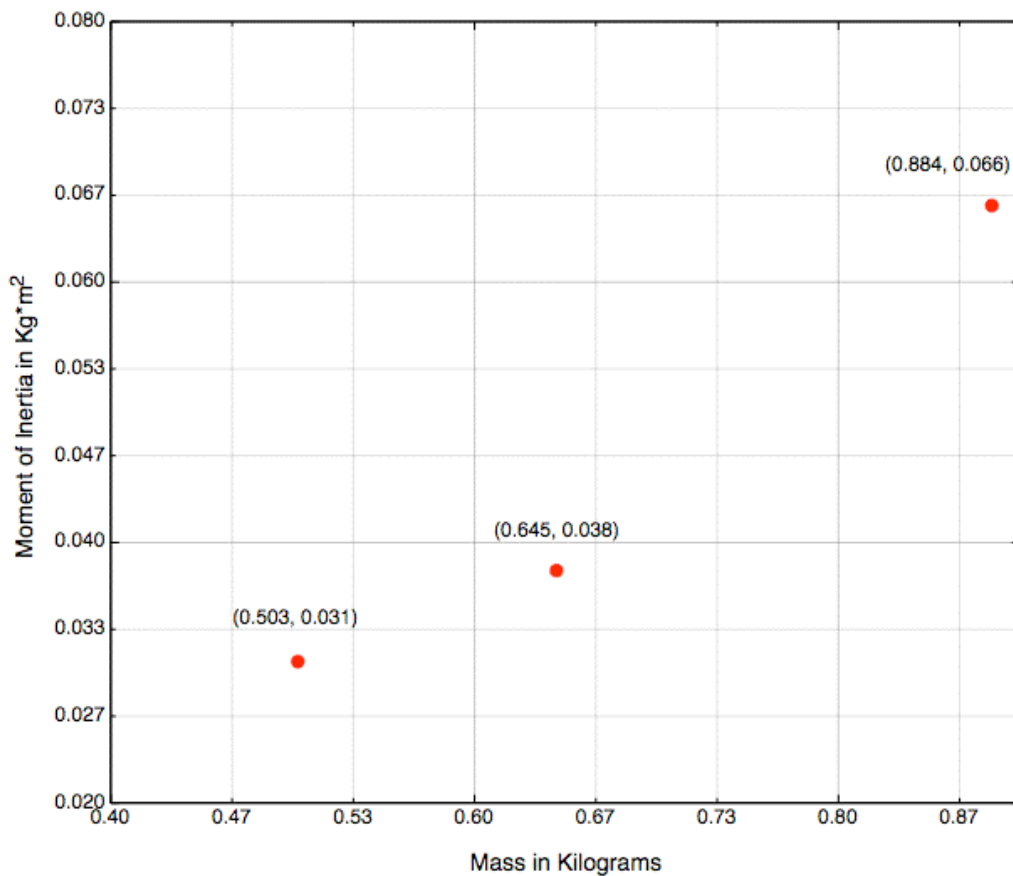


Figure 2 shows the average moment of inertia for three different bats: a Little League, high school, and college bat.

To determine the axial moment of inertia of the bats, it was assumed that the bats were a hollow, thin-walled cylinder. An object with this shape has a moment of inertia of:

$$I = mr^2 \quad (6)$$

Next, the kinematic properties of the bat and ball were determined. Notice that these are the independent variables of this investigation. The purpose of this investigation is to see how  $E$  and  $Psi$  change, as a function of bat velocity, as a player faces different pitch velocities, pitch angles, and angular velocities. A range of swing speeds from 20mph to 120mph was used for each different level of play so that results from each trial could be examined on a common axis. Hubbard *et al* determined their linear velocity, angular velocity, and pitch angle from experimental data they collected at the 1996 Atlanta Olympics for another study<sup>ix</sup>. Unfortunately, no such data exists for Little League or high school level players. Thus, the linear velocities for each level were approximated by using two typical fastball pitch speeds for each level of play. The angular velocity for each speed was scaled by the ratio of the angular velocity to linear velocity of the fastball used in Hubbard *et al*'s study. The pitch angle proved to be the most difficult variable to estimate because the speed of the ball crossing home plate is known while its direction is not. Thus the pitch angle was determined in Phase I of the optimization routine for each level of play.

## Computational Methods

Once the parameters and variables had been defined, the MATLAB program was modified accordingly. The optimization routine was run six times, using two different pitch speeds for each level of play. This routine can be broken into four different phases to better understand its inner workings.

To determine the pitch angle, Phase I was written and added to Hubbard *et al*'s optimization routine. Phase I "guessed" the pitch angle and initial velocity assuming that the ball crossed the center of the strike zone with a known final velocity and release height. The term "guessed" refers to the fact that the program calculated the final velocity of the ball many times until it found the initial conditions that resulted in the desired velocity. The release height and center of the strike zone were estimated by using the average player height of the age group from the Center for Disease Control. Note that a simple geometrical analysis could have given a rough estimate of the pitch angle, however it would not account for the lift and drag forces the pitched ball experienced. Phase I used functions defined in the flight analysis of the batted ball program in order to account for these forces. Phase I was run six times, once for each different pitch velocity.

Phase II of the program consisted of the bat-ball collision. This analysis assumes that the ball strikes the bat at the center of percussion of the bat. Thus there will be no impulsive reaction between the bat handle and batter's hands. In other words, the batter does not experience any "sting" of the bat. Furthermore, it is assumed that the bat is horizontal and perpendicular to the vertical plane of the pitched and batted ball. The impact analysis uses Stronge's<sup>x</sup> planar rigid-body impact methodology and terminology. It accounts for both the translational and rotational motion in order to determine the batted ball's velocity, angular velocity, and launch angle. Thus Phase II is given pitch velocity, angular velocity, pitch angle, bat speed,  $E$ , and  $Psi$ , and returns the kinematic properties of the ball after impact.

The results of the impact analysis then become the input variables for the flight simulation, or Phase III of the program. Phase III consists of several smaller programs that are each defined as functions. One function is responsible for calculating the drag coefficient of the ball. As stated earlier, the drag coefficient is dependent upon the speed of the ball. Because the speed of the ball is changing over time, the drag coefficient must be calculated repeatedly. The lift coefficient is also

calculated dynamically. These results are then passed to another function which actually returns the range of the batted ball.

The final phase of the program, Phase IV, is responsible for the optimization of  $E$  and  $Psi$ .  $E$  and  $Psi$  are an initial condition required by Phase II of the routine to calculate the initial flight conditions of the ball. Thus, the user must give an initial guess of the values of  $E$  and  $Psi$ . Phase IV uses the MATLAB defined function 'fminsearch' to determine what values of  $E$  and  $Psi$  will result in the largest range. Thus, after each completion of Phases II and III,  $E$  and  $Psi$  are modified and the program repeats. Each range returned is saved and 'fminsearch' compares each new range to the previous one to determine which is larger. After a many iterations, the  $E$  and  $Psi$  that resulted in the largest range are returned. A script was written in order to carry out this entire process for 40 different swing velocities and create plots of the results.

## Results and Discussion

The batted ball velocity and angular velocity are plotted in Figures 3 and 4. It is not surprising to find the velocity of the batted ball increases linearly as swing speed increases: the faster one swings the bat, the faster the exit velocity of the ball. The mass of the bat also plays a significant role in the resulting exit velocity of the ball. There is very little difference between exit velocities for the pitch speeds at the same level of play. For example, a college level player does not hit the ball with a higher exit velocity when facing a 90mph pitcher versus an 80mph pitcher. However, there is a significant difference in the ball exit velocities between different levels of play. Recall, the main difference between play levels is the size of the bat used. Thus, heavier bats result in higher exit velocity for the same swing speed. While the angular velocity of the ball does not have a linear dependence on swing speed, it does increase as swing speed increases. Unlike linear velocity, the angular velocity does have a dependence upon the initial pitch conditions. The curves are not grouped in pairs as before. This indicates that bat size, initial pitch angular velocity, and swing speed all affect the exit ball angular velocity.

Another interesting result seen in Figure 4 is the angular velocities found in Hubbard *et al's* study are less than the angular velocities of college level players. Because Major League players face higher pitch velocities, one would expect them to produce higher exit ball angular velocities. However, Major League players use wooden bats which have a different axial moment of inertia. Recall that an aluminum bat's axial moment of inertia is equal to the mass times radius squared while wooden bats have a moment of inertia equal to one half its mass times radius. Thus aluminum bats are able to create more angular velocity when they hit the ball.

This result explains why college level players are able hit the ball slightly farther than Major League players even though Major Leaguers face faster pitch speeds. Because college level players can hit balls with higher angular velocities, their balls have more lift force. Thus they remain in the air longer and can travel farther. This is illustrated in Figure 5 which is a plot of range versus bat velocity. Again, as swing speed increases, so does range. The curves in this plot also group according to play level suggesting that the difference in bat is more important than the difference in pitch speed.

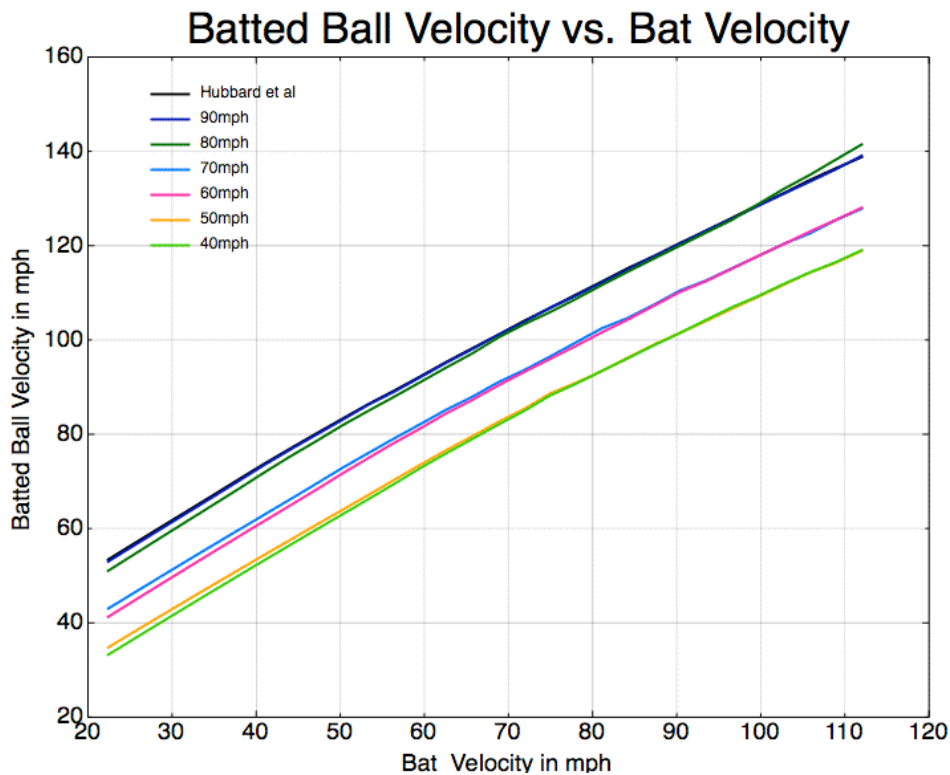


Figure 3: Shows batted ball velocity as a function of bat speed. As intuition suggests, batted ball velocity increases linearly with bat velocity. The grouping of the curves according to bat speed suggests that bat size affects batted ball velocity more than initial pitch speed.

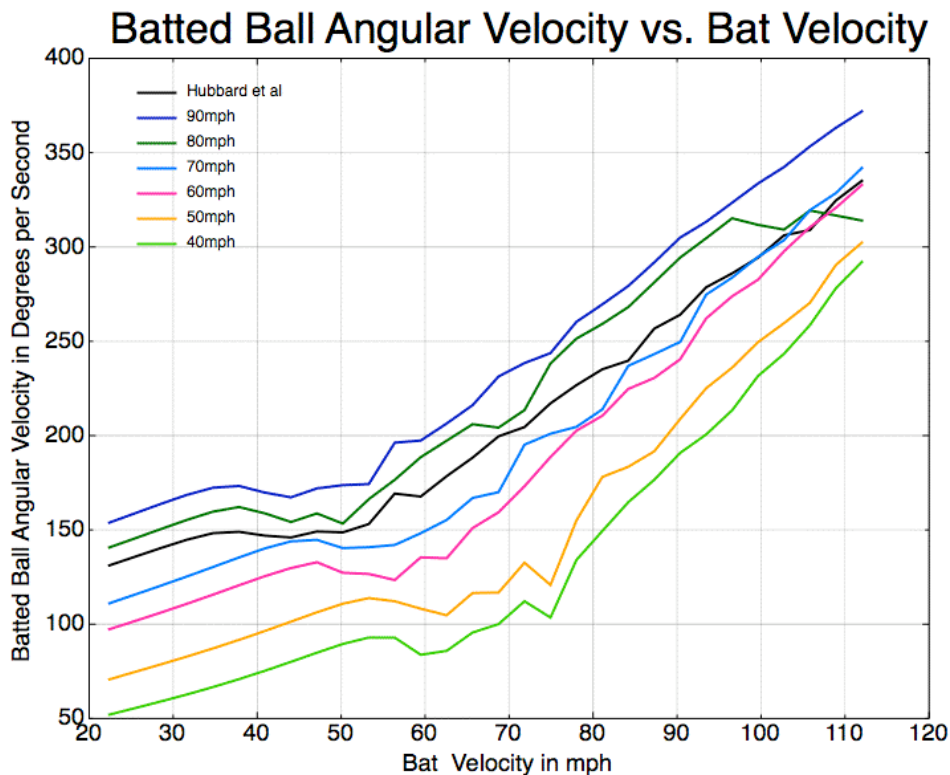


Figure 4: Like the batted ball velocity, batted ball angular velocity increases as bat velocity increases. Notice that the Major League data is below the college league data. This is due to the difference between wooden and aluminum bats.

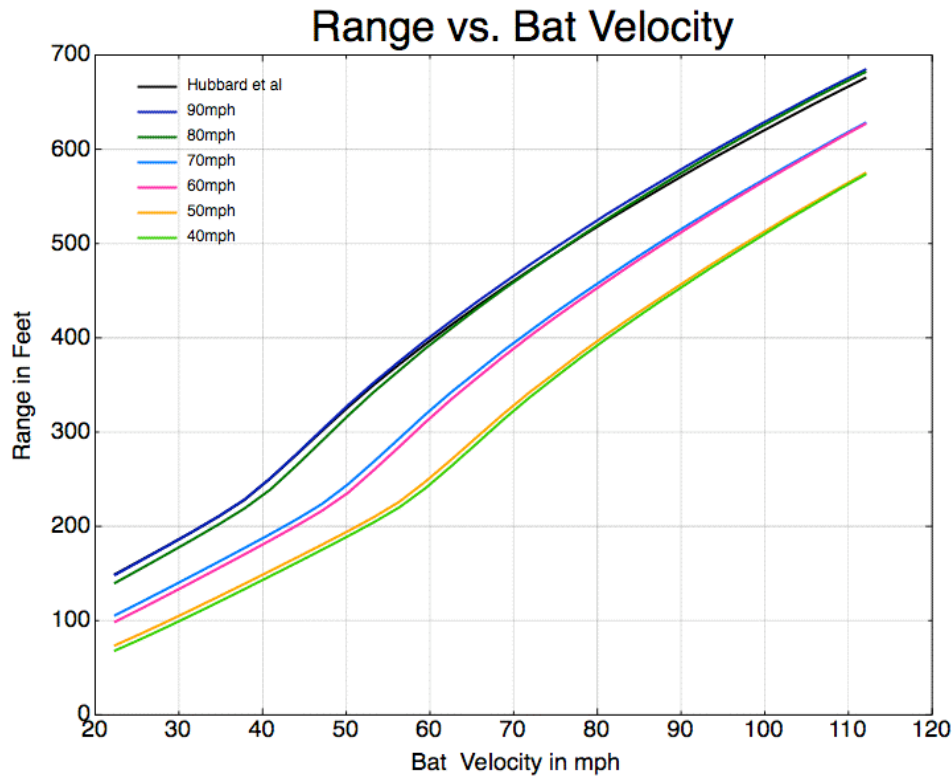


Figure 5: Shows range as a function of bat velocity. As bat velocity increases, so does the range. The curves are grouped similarly to Figure 3.

Figures 6 and 7 are plots of the optimum undercut angle,  $\Psi$ , and undercut distance,  $E$  respectively. From Figure 6, it is clear that as players face faster pitch speeds and use heavier bats, they must swing with a smaller upward angle than lower level players. From the opposite perspective, Little League level players must swing up on the ball more than Major League players. This is consistent with Figure 7 which shows that as pitch velocity and bat weight increase, the under cut distance becomes smaller. The shapes of the curves in both Figure 6 and 7 are difficult to classify. It is important to keep in mind that there are three input variables and two parameters that are being optimized in this study. Thus it is difficult to attribute bumps in the graph to one variable or another. However, the green curve in Figure 6 does flatten out extremely in the high swing speeds. This is probably due to a rounding error resulting in a variable becoming undefined at such extreme swing speeds.

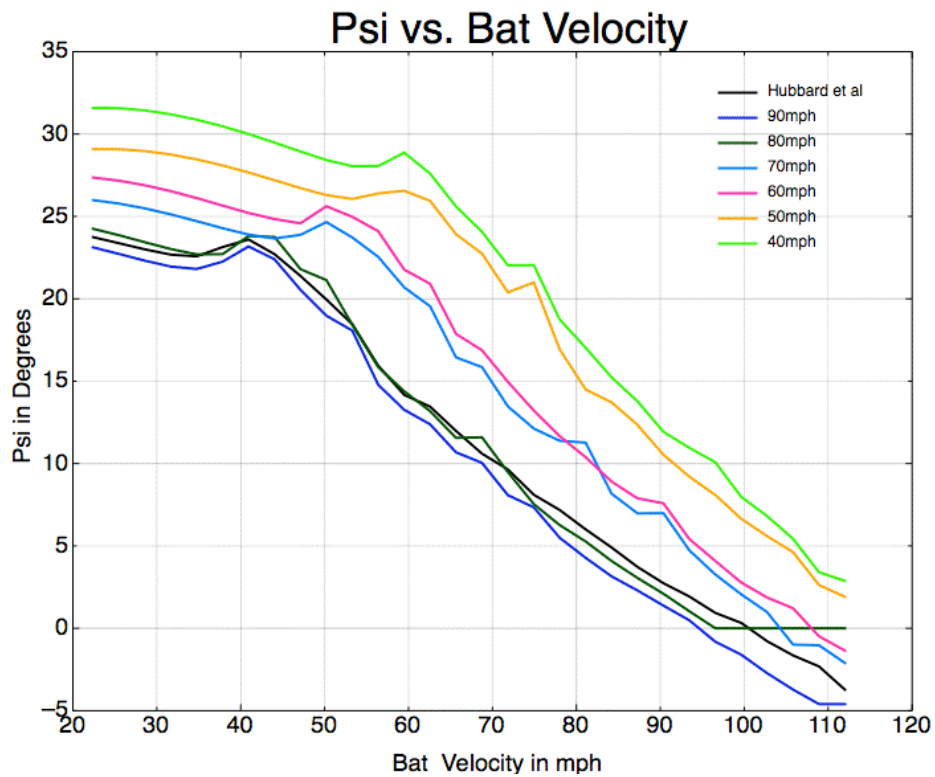


Figure 6: Undercut angle is graphed as a function of bat velocity. As bat velocity increases, undercut angle generally decreases. Undercut angle also decreases as initial pitch speed and bat weight increases.

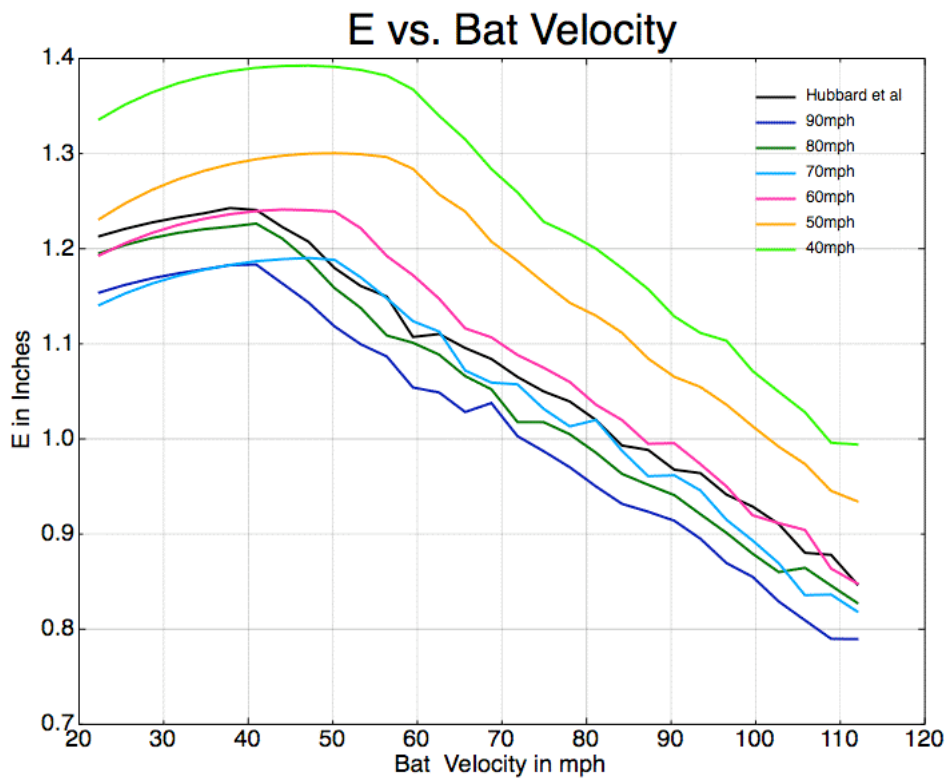


Figure 7: Undercut distance is graphed as a function of bat velocity. After increasing slightly, undercut distance decreases linearly with bat velocity. As initial pitch speed and bat weight increase, the undercut distance decreases. This is consistent with Figure 6.

## Conclusions and Future Work

The purpose of this study was to determine how the optimum swing strategy changed for different levels of play. The results presented show the following:

1. There is an optimal strategy for achieving maximum range. At a given bat speed, there is a specific  $E$  and  $Psi$  with which the bat should approach the ball.
2. There is a significant difference in the optimal strategy for different levels of play. Generally, as players move up in levels of play, the  $Psi$  and  $E$  values of their optimal swing will decrease. For example, Little League players should swing “up” on the ball more than a high school level player.
3. This study has not found the optimal strategy for being a “successful” hitter. Different teams, coaches, and players all have different perspectives as to what a hitter should accomplish. Furthermore, this investigation assumed that the player has the ability to choose where contact between the bat and ball occurs. This is not within the ability of all players.

As discussed earlier, the main differences between different levels of play in baseball are the bat and dynamic properties of the ball. Thus this investigation only modified the bat and dynamic ball properties. It would be interesting to see how the results would change for the sport of softball because of the larger ball and underhand pitch delivery. Many softball coaches have played baseball and take the strategies from one game to next. However, due to the differences enumerated, it is likely that the optimal swing strategies of baseball and softball are not the same.

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